Combinatorial Arabesque: A new concept

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ABSTRACT

This paper is interested in developing a new application in combinatorial analysis, which will be called "combinatorial arabesque". There are many disciplines, which have a relationship with analytic combinatorics such as "Graph theory", "combinatorial optimization" and "Probabilistic combinatorics". However, applying combinatorial analysis on geometric patterns is actually a new topic and there is no previous study on this subject. This topic seems to be a simple composition of geometrical patterns, but it really uses many branches of mathematics such as counting techniques, analysis, and structural algebra related to geometric transformations. The results of this paper is two tenors, the first is mathematical by determining all of the specific kind of motifs that shall be called "Entirely symmetrical motifs". The second tenor is artistic and decorative by constructing beautiful motif from ugly one. Finally, a mathematical definition of beauty degree of a motif is given.

Key Words: Combinatorial analysis, Invariant group, Motif.

INTRODUCTION

Many scientists were interested in the arabesque and investigated this art from mathematical point of view. To the best of our knowledge, the first scientist who was interested in motifs composed only by diagonally bi-colored squares is Sébastien Truchet, a French mathematician, topographer, and engineer born in 1657 and died in 1729. These kind of motifs bear his name and are called "Truchet tiles" (Fig. 1). The work on bi-colored squares is published in 1704 and a digitized version can be found in Truchet (1704). After few years, precisely in 1722, Dominique Douat takes over the study done by Truchet and published a book, illustrating simple examples of counting via combinations and permutations, for more details see Dominique (1722). The complete works of Sébastien Truchet is recapitulated in André and Denis (1999).

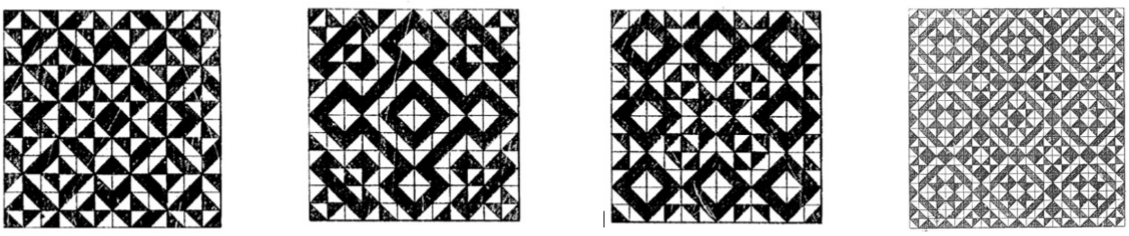


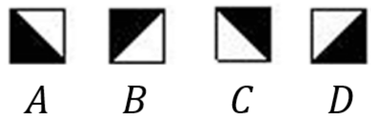
Figure 1: Some examples of Truchet tiles

Regarding recent works, Smith (1987) proposed alternatives for Truchet tiles with only two possible orientations instead of four, resulting to binary coding. Lord and Ranganathan (2006) gave three-dimensional generalizations. Reimann (2009) developed techniques to create text by mean of Truchet tiles having two circular arcs. Krawczyk (2011) extended the bi-colored square to other shapes, not necessarily separated in color at the diagonal. The attention paid to geometric patterns in general, and especially Islamic ones, has different issues. Broug (2019) explains in details the traditional ruler and compass constructions. Lu and Steinhardt (2007) studied the design of Islamic geometric patterns by means of a set of five tiles, called Girih tiles. Many authors developed software that generate geometrical designs, see among others Kaplan (2000), Kaplan (2005), Al-Darwich (2011), and Sarhangi (2012). Our interest in studying arabesques, based on bicolored tiles, is purely mathematical. An equivalence relation on the set of all Truchet tiles having the same dimension is defined. The motifs entirely symmetric will be characterized then explicitly determined. Even the beauty notion, it will be investigated by mathematical tools as isometric transformations and invariance groups.

The rest of this paper is organized as follows. First, a description of the mathematical model of our arabesque is given with the terminology distinguishing between, motif, pattern, and arabesque. This is followed by enumerating all arabesques that are entirely symmetric. It will be seen that the property of "entirely symmetric" is not necessarily a synonymous of beauty. Then, an investigation on "how to measure the beauty degree" of an arabesque will be accomplished. It is worth mentioning that, this paper is part of master thesis of Al-haboob (2018), that studied combinatorial arabesque in many aspects.

MATERIALS AND METHODS

The basic element used in these arabesques is a square divided diagonally into two isometric triangles colored differently, which will be called diagonally bicolored square. This square has four possible orientations obtained by successive rotation of angle $\pi/2$. Truchet notation "*A,B,C,D*" will be used to designate these four positions.



we observe that $\hat{A}=C$ & $\hat{B}=D$, where the transformation $\hat{\bullet}$ stands for the central symmetry, centered at the south-east corner of the square. Each composition of n^2 diagonally bicolored squares having n rows and n columns is called motif of dimension n. Figure 2 gives some examples of motifs of different dimensions.

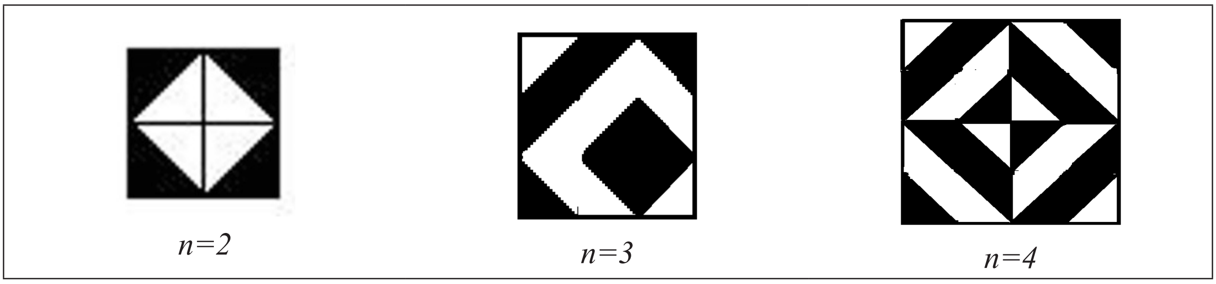


Figure2: Motifs of different dimensions

Each motif of dimension n can be expressed as a matrix $M_n = (\chi_{ij})_{1\le i,j\le n}$, where the entries χ_{ij} are in the set $\{A,B,C,D\}$. So, the number of n-dimensional motifs is equal to 4^{n^2}. We call pattern associated to motif M_n, the picture obtained by concatenating beside M_n the same motif M_n and below the central symmetry motif M_n:

$$P^{(n)} = \begin{pmatrix} M_n & M_n \\ \widehat{M}_n & \widehat{M}_n \end{pmatrix}$$

For example, if $n=2$, the motif M_n will be equal to, $M_2 = \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix}$ therefore the associated pattern will be as follows:

x_{11}	x_{12}
x_{21}	x_{22}

Motif M_2

	x_{11}	x_{12}	x_{11}	x_{12}
	x_{21}	x_{22}	x_{21}	x_{22}
$\hat{\bullet}$	$\hat{x}_{22}$	$\hat{x}_{21}$	$\hat{x}_{22}$	$\hat{x}_{21}$
	$\hat{x}_{12}$	$\hat{x}_{11}$	$\hat{x}_{12}$	$\hat{x}_{11}$

Pattern $P^{(2)}$ related to M_2

The elements $\hat{x}_{ij}$ represent the central symmetry, centered at the center of the pattern square, of χ_{ij}. It is easily proved that the transformation $\hat{}$ fulfills the following:

$$M_n = \begin{pmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nn} \end{pmatrix} \Rightarrow \widehat{M}_n = \begin{pmatrix} \hat{x}_{nn} & \cdots & \hat{x}_{n1} \\ \vdots & \ddots & \vdots \\ \hat{x}_{1n} & \cdots & \hat{x}_{11} \end{pmatrix}$$

Equivalently:

$$M_n = \left(x_{i,j}\right)_{1\le i,j\le n} \Rightarrow \widehat{M}_n = \left(\hat{x}_{n-i+1,n-j+1}\right)_{1\le i,j\le n}$$

The image resulting from paving the pattern associated to a motif M_n is called arabesque related to M_n. Figure 3 explains the progression from motif, to pattern to reach finally the related arabesque:

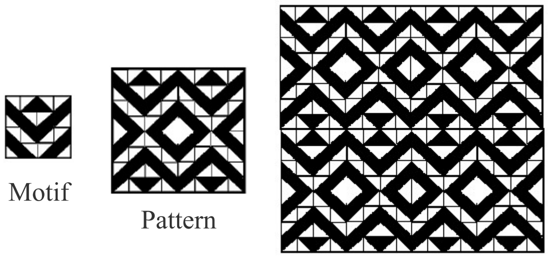


Figure 3: Example of Motif of dimension $n = 4$ and the related Pattern and Arabesque

Two different motifs may lead to the same arabesque; figure 4 explains this finding:

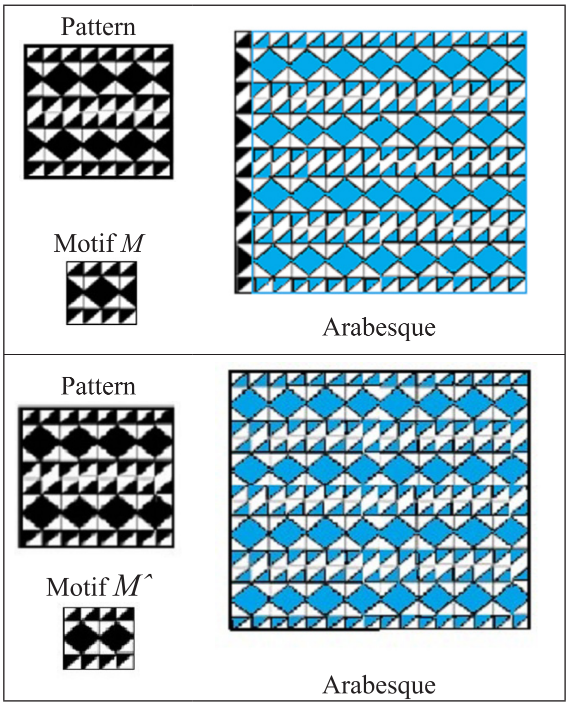


Figure 4: Two different motifs generating the same arabesque

For the arabesque, the paving is supposed to be made up to infinity; there is no border for an arabesque. In the figure 4, it is clear that the final arabesque is identical for the two different motifs M and $\acute{M}$. This ascertainment leads to define an equivalence relation on the set of all motifs of same dimension.

Equivalence relation on arabesques:

Definition 1: Let M and $\acute{M}$ be two motifs of same dimension. It is said that M is in relation with $\acute{M}$, and let write $M \;\mathcal{R}\; \acute{M}$, if and only if the arabesque related to M is identical to the arabesque related to $\acute{M}$.

Let M_n be a motif of dimension n. It is obvious that the symmetrical motif $\widehat{M}_n$ belongs always to the class $\overline{M_n}$, this latter is defined by:

$$\overline{M_n} = \{\acute{M}_n \in \mathcal{M}_n \mid M_n \;\mathcal{R}\; \acute{M}_n\},$$

Where $\mathcal{M}_n$ stands for the set of all motifs of dimension. It happens that $\overline{M_n} = M_n$. The motifs that belong to $\overline{M_n}$ can be extracted from the arabesque related to M_n. For this purpose, let $P_{ij}^{(n)}$ denotes the motif of dimension n starting from the point of coordinates (i,j). In that way, the motif $P_{00}^{(n)}$ coincides with M_n and $P_{n0}^{(n)}$, also $P_{nn}^{(n)}$ is exactly $\widehat{M}_n$. Figure 5 gives an example for n=2, where the motifs $P_{01}^{(2)}$ and its central symmetrical $P_{23}^{(2)}$ are represented by bold lines. The bullet indicates the starting point (i,j). For Figure 4, one can observe that the motif $\acute{M}$ is none other than $P_{01}^{(4)}$.

x_{11}	x_{12}	x_{11}	x_{12}	x_{11}	x_{12}
x_{21}	x_{22}	x_{21}	x_{22}	x_{21}	x_{22}
$\hat{x}_{22}$	$\hat{x}_{21}$	$\hat{x}_{22}$	$\hat{x}_{21}$	$\hat{x}_{22}$	$\hat{x}_{21}$
$\hat{x}_{12}$	$\hat{x}_{11}$	$\hat{x}_{12}$	$\hat{x}_{11}$	$\hat{x}_{12}$	$\hat{x}_{11}$
x_{11}	x_{12}	x_{11}	x_{12}	x_{11}	x_{12}
x_{21}	x_{22}	x_{21}	x_{22}	x_{21}	x_{22}

Figure 5: Arabesque of M_2

Theorem 1:

$$P_{ij}^{(n)} \in \overline{M_n} \Leftrightarrow P_{n+i,n+j}^{(n)} = \hat{P}_{ij}^{(n)}$$

Proof: It is sufficient to note that the class membership of $P_{ij}^{(n)}$ is equivalent to the fact that the motif of dimension $2n$ starting from the point of coordinates (i,j) must be the pattern associated to $P_{ij}^{(n)}$.

■

Henceforth, this relation namely $P_{n+i,n+j}^{(n)} = \hat{P}_{ij}^{(n)}$ is called: "Central Symmetry Condition" in short (CSC).

Entirely symmetrical motifs:

Symmetry is very important in Islamic motifs. In the last section, it has been proved that the central symmetry condition is equivalent to the belonging to the class. The questions that arise are threefold:

- Is it possible to find a motif M_n where all motifs $P_{ij}^{(n)}$; $0 \leq i,j \leq -1$; extracted from the arabesque associated to M_n satisfy (CSC)?
- Once the existence of entirely symmetrical motifs proved, how could identify them and then determine all of them?
- Does the property of entirely symmetric generates necessarily beautiful arabesques?

The object of this section is to answer all these questions.

Definition 2: A motif M_n is called "Entirely symmetrical motif" if all motifs $P_{ij}^{(n)}$; $0 \leq i,j \leq n-1$; extracted from the arabesque associated to M_n satisfy (CSC**).**

In order to simplify the mathematical expressions, the following notations are adopted:

$$\begin{pmatrix} x_{11} & x_{11} & \cdot & \cdot & x_{11} \\ x_{21} & x_{21} & \cdot & \cdot & x_{21} \\ x_{31} & x_{31} & \cdot & \cdot & x_{31} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ x_{n1} & x_{n1} & \cdot & \cdot & x_{n1} \end{pmatrix} \triangleq \begin{pmatrix} x_{11} \\ x_{21} \\ x_{31} \\ \cdot \\ \cdot \\ \cdot \\ x_{n1} \end{pmatrix} \odot n$$

and

$$\begin{pmatrix} x_{11} & x_{12} & \cdot & \cdot & x_{1n} \\ x_{11} & x_{12} & \cdot & \cdot & x_{1n} \\ x_{11} & x_{12} & \cdot & \cdot & x_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ x_{11} & x_{12} & \cdot & \cdot & x_{1n} \end{pmatrix} \triangleq (x_{11} \quad x_{12} \quad \cdot \quad \cdot \quad x_{1n}) \odot n$$

<u>Lemma 1:</u> Let $M_n = (C_1 \quad C_2 \quad \cdots \quad C_3)$ be a motif of dimension n, where C_i stands for the i^{th} column of M_n, the following holds:

1. If n is even, then

$$P_{01}^{(n)} \in \overline{M_n} \Leftrightarrow M_n = (C_1 \quad C_2) \odot \frac{n}{2}$$

2. If n is odd, then

$$P_{01}^{(n)} \in \overline{M_n} \Leftrightarrow M_n = (C_1) \odot n$$

Proof: According to Theorem 1:

$$P_{01}^{(n)} \in \overline{M_n} \Leftrightarrow P_{n,n+1}^{(n)} = \hat{P}_{01}^{(n)}$$

The pattern associated to the motif $M_n = (C_1 \quad C_2 \cdots C_n)$ concatenated with the first column of M_n is exactly:

C_1	C_2	$\cdots$	C_n	C_1	C_2	$\cdots$	C_n	C_1
$\hat{C}_n$	$\hat{C}_{n-1}$	$\cdots$	$\hat{C}_1$	$\hat{C}_n$	$\hat{C}_{n-1}$	$\cdots$	$\hat{C}_1$	$\hat{C}_n$

The motif $P_{01}^{(n)}$ and its central symmetry, centered at point $(n,n + 1)$, are represented by bold lines. It should be made clear that the scheme above is a part of the arabesque related to the motif M_n. The condition $P_{n,n+1}^{(n)} = \hat{P}_{01}^{(n)}$ is equivalent to:

$$\begin{cases} C_{n-1} = C_1 \\ C_i = C_{i+2}, \ \forall i = 1, \dots, n-2 \\ C_n = C_2 \end{cases}$$

These conditions can be lumped into one identity:

$$C_i = C_{i+2}, \ \forall i = 1, \dots, n$$

On the identity above, when the index $i + 2$ exceeds n, it is replaced by the integer $(i + 2) \bmod (n)$. In the case where the dimension n is odd, the relations $C_i = C_{i+2}$ for all indices i mean that all the columns of the motif M_n are identical:

When n is even, the same relations mean that the two first column are duplicated $n/2$ times:

$$P_{01}^{(n)} \in \overline{M_n} \Leftrightarrow M_n = (C_1 \quad C_2) \odot \frac{n}{2}$$

■

The belonging effect of $P_{10}^{(n)}$ to the class is studied in the next lemma.

<u>Lemma 2:</u> Let $M_n = \begin{pmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{pmatrix}$ be a motif of dimension n, where R_i stands for the i^{th} row of M_n, the following holds:

1. If n is even, then

$$P_{10}^{(n)} \in \overline{M_n} \Leftrightarrow M_n = \begin{pmatrix} R_1 \\ \hat{R}_1 \end{pmatrix} \odot \frac{n}{2}$$

2. If is odd, then

$$P_{10}^{(n)} \in \overline{M_n} \Leftrightarrow M_n = \begin{pmatrix} \begin{pmatrix} R_1 \\ \hat{R}_1 \end{pmatrix} \odot \frac{n-1}{2} \\ R_1 \end{pmatrix}$$

■

Proof: The proof follows the same demarche adopted in Lemma 1.

Motifs of odd dimension:

Theorem 2: Let n be an odd number and $M_n = (\chi_{ij})_{1\leq i,j\leq n}$ be a motif of dimension n (where $\chi_{ij} \in \{A,B,C,D\}$), we have:

$$M_n \text{is entirely symmetric} \Leftrightarrow M_n = \begin{pmatrix} x_{11} \\ \hat{x}_{11} \\ x_{11} \\ \cdot \\ \cdot \\ \hat{x}_{11} \\ x_{11} \end{pmatrix} \odot n$$

<u>Proof:</u> Let M_n be an entirely symmetric motif. According to Definition 2, all motifs $P_{ij}^{(n)}$; $0 \leq i,j \leq n-1$; extracted from the arabesque associated to M_n satisfy (CSC), in particular $P_{01}^{(n)}$ and $P_{10}^{(n)}$. Since n is odd, according to Lemma 1 and Lemma 2, it follows that:

$$\begin{cases} P_{01}^{(n)} \in \overline{M_n} \\ P_{10}^{(n)} \in \overline{M_n} \end{cases} \Rightarrow \begin{cases} M_n = (C_1) \odot n \\ M_n = \begin{pmatrix} \begin{pmatrix} R_1 \\ \hat{R}_1 \end{pmatrix} \odot \frac{n-1}{2} \\ R_1 \end{pmatrix} \end{cases}$$

The first condition means that all column of M_n are identical and the second one expresses that their rows alternate between R_1 and its central symmetry $\acute{R}_1$, so the motif M_n has necessarily the following form:

$$M_n = \begin{pmatrix} x_{11} \\ \hat{x}_{11} \\ x_{11} \\ \cdot \\ \cdot \\ \hat{x}_{11} \\ x_{11} \end{pmatrix} \odot n$$

It is easy to prove that this form is really an entirely symmetrical motif.

■

Remark1: For the case "n odd", the motifs entirely symmetrical admit only one variable, which is χ_{11}, that takes four values corresponding to the basic diagonally bi-colored squares. Figure 6 gives the four different patterns and the related arabesques. It is remarkable that the four motifs generate two different arabesques.

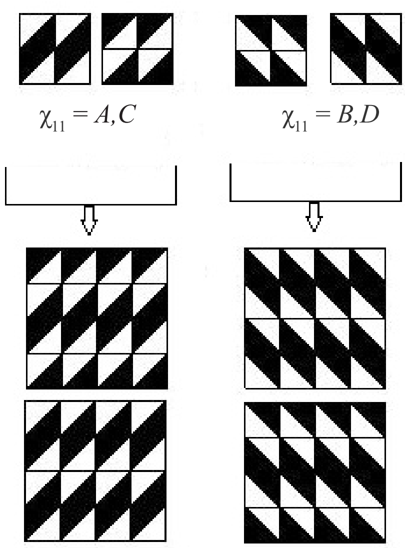


Figure 6: Odd motifs entirely symmetric and the related arabesques

Motifs of even dimension:

Theorem 3: Let n be an even number and $M_n = (\chi_{ij})_{1\le i,j\le n}$ be a motif of dimension n (where $\chi_{ij} \in \{A,B,C,D\}$), we have:

$$M_n \text{ is entirely symmetric } \Leftrightarrow M_n = \begin{pmatrix} \begin{pmatrix} x_{11} & x_{12} \\ \hat{x}_{12} & \hat{x}_{11} \end{pmatrix} \\ \circledcirc \frac{n}{2} \end{pmatrix} \circledcirc \frac{n}{2}$$

Proof: The proof can be achieved easily by using Lemma 1and Lemma 2.

■

Remark 2: Each motif $M_2 = \begin{pmatrix} x_{11} & x_{12} \\ \hat{x}_{12} & \hat{x}_{11} \end{pmatrix}$ depends on the variables χ_{11} and χ_{12}, these two variables vary in the set $\{A,B,C,D\}$. So, the number of motifs of dimension 2 that are entirely symmetric is equal to 16, they generate 6 different arabesques (see Figure 7).

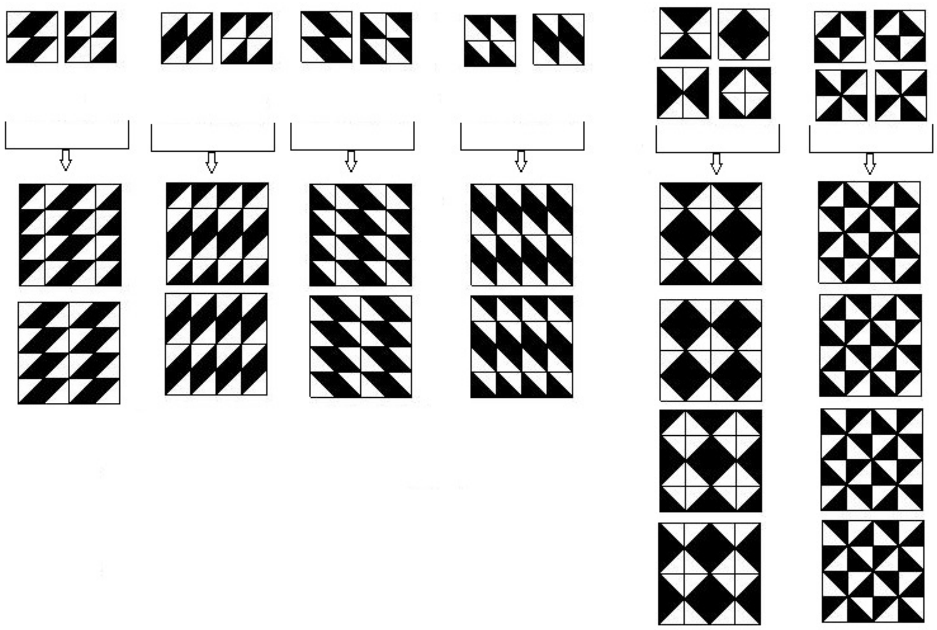


Figure 7: At the top the different motifs $M_2 = \begin{pmatrix} x_{11} & x_{12} \\ \hat{x}_{12} & \hat{x}_{11} \end{pmatrix}$ and at the bottom the associated arabesques

It is useful to note that the figure 7, corresponding to the case "n even", includes the motifs entirely symmetrical already obtained for the previous case "n odd". In fact, the odd case corresponds to the motif $M_2 = \begin{pmatrix} x_{11} & x_{12} \\ \hat{x}_{12} & \hat{x}_{11} \end{pmatrix}$ satisfying $\chi_{12} = \chi_{11}$.

Looking into all arabesques entirely symmetric, shown in Figure 7, one can deduce that the property of "entirely symmetrical" is not a synonym of extreme beauty. Figure 8 gives an example of arabesque that does not satisfy the condition of "entirely symmetric", but it seems more beautiful than all arabesques entirely symmetric.

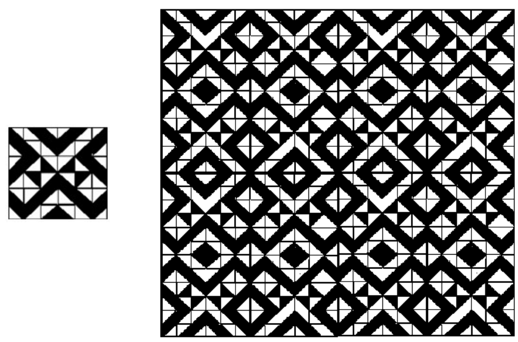


Figure 8: On the left a motif not entirely symmetric and on the right the related arabesque

BEAUTY ANALYSIS

Invariant group:

The group of geometrical transformations keeping invariant a motif is denoted commonly by G, and the internal composition law is the composition of transformations, that is denoted ○. The geometrical transformations that can be involved for any motifs are:

- Identity, denoted by *I*.
- Rotation of angle π/2 centered at the center of the motif, denoted *R*. If $R \in \boldsymbol{G}$, then $R^{-1} \in \boldsymbol{G}$.
- Central symmetry centered at the center of the considered motif, denoted by *S*. Note that $R^2 = S$.
- Reflection in the vertical axis passing through the motif center, denoted by S_v.
- Reflection in the horizontal axis passing through the motif center, denoted by S_h.
- Reflection in the first diagonal axis of the motif, denoted by S_1.
- Reflection in the second diagonal axis of the motif, denoted by S_2.

The composition of two distinct reflections from the ones mentioned above is a rotation, and the composition between rotation and reflection is a reflection. Also, it is useful to recall that for any reflection *s*, we have $s^2 \triangleq s \circ s = I$. It could be proved that the set

$\boldsymbol{G}_m = \{I,R,R^{-1},S,S_v,S_h,S_1,S_2\}$

is a group, it is indeed the "Dihedral group" D_8 of order 8.

Definition 3: The beauty degree of an arabesque is the order of the group keeping invariant the related equivalence class. The beauty degree is said maximal, if the order of the group is G_m.

<u>Example 1:</u> A motif of dimension n =4.

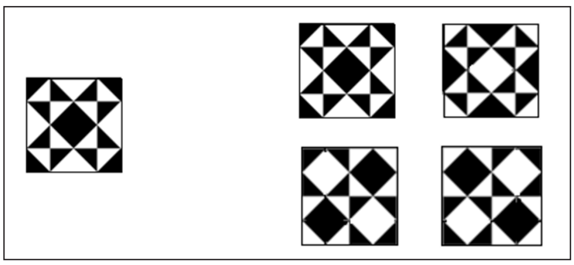


Figure 9:Motif of dimension n=4 and the related class

$$G = G_m \qquad \text{ord}\,(G) = 8$$

For the two last motifs in Figure 9, we remark that they symmetrical, each one is the image of the other by S_v and alsoby S_h. So, the group keeping the class invariant is actually $\boldsymbol{G_m}$.

Figure 10 illustrates the commune arabesque for all motifs appearing in Figure 9:

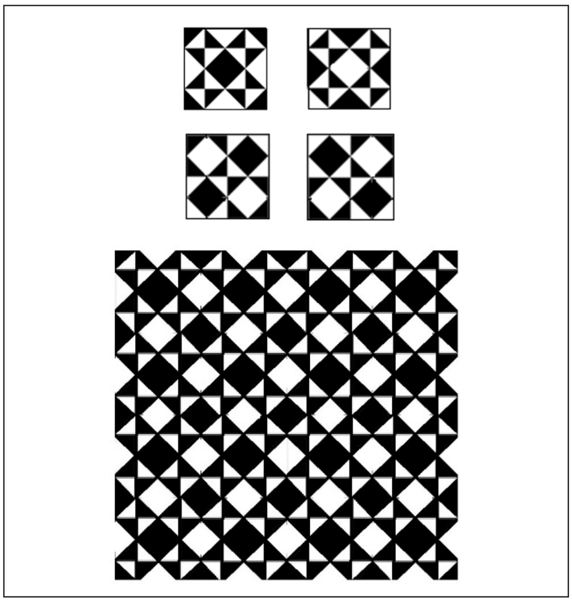


Figure 10:The commune arabesque for all motifs of figure 9.

<u>Example 2:</u> Figure 11 illustrates an example where the group for each motif is $\boldsymbol{G} = \{I, S_v\}$ whereas the group for all the class is $\boldsymbol{G} = \{I, S_v, S_h, S\}$.

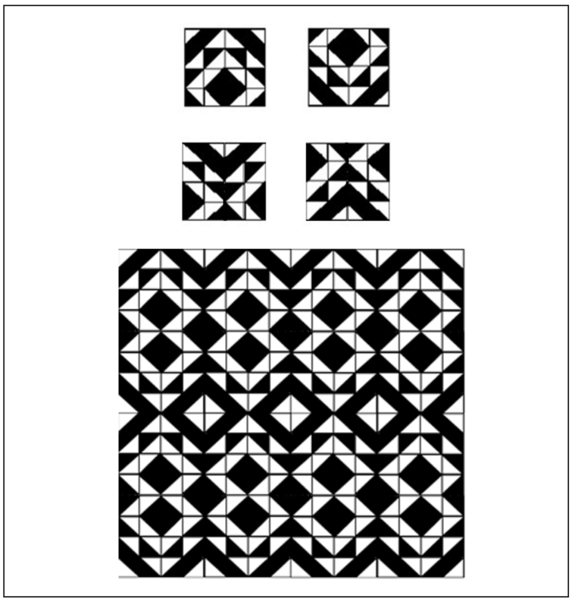


Figure 11: Four motifs belonging to the same class and its commune arabesque.

It is clear that the arabesque in Example 1 is more beautiful than the one in Example

2. The order of invariance group is 8 for Example 1, and it is only 4 for Example 2.

Construct beautiful arabesques

The group of invariants can be reduced to one element only, the identity. In this case, the arabesque is not beautiful. In this section, it will be seen how to construct from this kind of motifs other ones larger in dimension but mostly more beautiful.

In Figure 12, motif has no symmetry; the group of invariants is precisely $\boldsymbol{G} = \{I,\}$

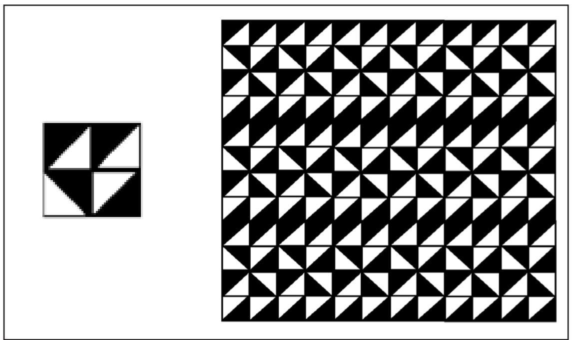


Figure 12:Motif without symmetry and its related arabesque

This arabesque seems not beautiful, due to the fact that the group cardinality is the smallest one. The idea consists in considering for example the Truchet pattern as a motif, which corresponds to a right concatenation of the motif reflection and then a central symmetry application:

$$TP^{(n)} = \begin{pmatrix} M_n & \underline{M_n} \\ \underline{\widehat{M}_n} & \widehat{M}_n \end{pmatrix}$$

The Truchet pattern associated to the motif above is shown in Figure13.

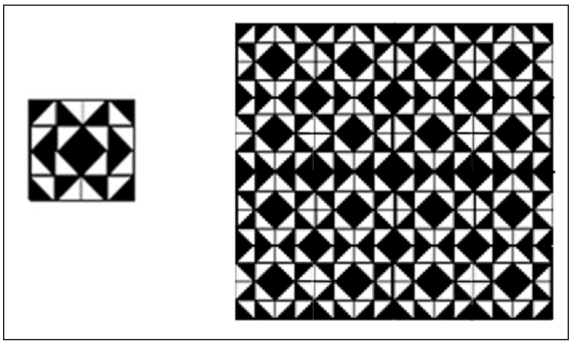


Figure 13:.Truchet pattern associated to the motif of figure 12 and its arabesque

The group of invariants for the constructed motif in Figure 13 is nothing else than $\boldsymbol{G} = \{I, S_v, S_h, S\}$. In that way, the group order grows from 1 to 4. By comparing the arabesques in figure 12 and figure 13, the difference of beauty degree is unequivocal.

CONCLUSION

In this paper, a new topic in combinatorics field, that is called "combinatorial arabesque", has been tackled. At first glance, this subject seems to be a simple combination of diagonally bi-colored squares, but it was found that it spans many mathematical disciplines. The outcomes of this work can be recapitulated in the following items:

- Give the mathematical model of the problem, in that way, all results have been dealt with mathematical objects and not with graphical ones.
- Determine and enumerate all motifs entirely symmetrical.
- Measure and define the beauty degree by means groups of transformations and invariance.
- Propose tricks allowing to produce beautiful arabesques.

As perspective of this work, it can be cited the following:

- Generalize this study for squares not only diagonally bi-colored but also equally (in surface) bi-colored, or also multicolored squares.
- Develop a software approach for generating beautiful Truchet tiles. A geometric pattern generator can be useful in architectural design or also in textile manufacturing.
- As one of the practical applications for this work, use the CNC Router machine to introduce the arabesque on any choices material then applied them in decoration.

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الملخص

هذه المقالة تستحدث تطبيقا جديدا في التوافقية التحليلية بمسمى الأرابسك التركيبي. توجد العديد من التطبيقات أو المجالات المتصلة بالتحليل التوافقي (أو التركيبي) منها على سبيل المثال لا الحصر نظرية الرسومات والإمثال التركيبي والاحتمالات التركيبية. لكن تطبيق التوافقية التحليلية على الزخارف الهندسية هو مجال لم يسبق دراسته أو التطرق إليه من قبل. هذا المجال لا يتعدى في ظاهره حدود التركيبات الزخرفية ولكنه في الواقع يستخدم العديد من الفروع في الرياضيات منها طرق العد والتحليل وكذلك الجبر الهيكلي المتعلق بالتحويلات الهندسية.

نتائج هذه الدراسة لها بعدان، الأول رياضي ويتمثل في تحديد ومن ثم تعداد جميع الزخارف من نوع خاص أسميناها «الزخارف المتناظرة كليًا». البعد الثاني هو فني زخرفي حيث تمكن هذه الدراسة من إنشاء تصاميم زخرفية في غاية الجمال من زخارف غير جميلة وإعطاء تعريف رياضي يقيس درجة الجمال لكل زخرفة.

الكلمات المفتاحية: التحليل التوافقي، زخارف، زمرة الثبات.