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Flipped Learning of Irrational Numbers: Saudi Arabia

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ABSTRACT

There is a growing interest in the flipped design model in K–12 mathematics classrooms, as it has been shown to have a positive influence on learning. This study aims to help improve students' learning of irrational numbers using the flipped design model. This study outlines four design elements of the flipped environment that are shown to increase student engagement. Two eighth-grade classes (n = 60) were examined using a quasi-experimental research design in terms of students' mathematical achievements and engagement levels. Each class was tested using a different instructional approach: traditional learning or flipped learning. The results indicated that the eighth-grade students in the flipped learning model group demonstrated an overall medium to a slightly high level of cognitive, behavioral, emotional, and social engagement, as well as a slightly high performance with irrational numbers. Also, when comparing the flipped and traditional learning environments, the results revealed that neither group was significantly different in terms of mathematics performance or engagement. Several recommendations and implications are discussed for teaching complex mathematics concepts via a flipped learning environment, including enriching the learning environment with student collaboration, social and emotional support, and problem solving.

KEYWORDS

Mathematics engagement, cognitive engagement, social engagement, behavioral engagement, student-centered, deep learning

CITATION

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1. Introduction

During the COVID-19 pandemic, mathematics education worldwide was dramatically affected due to the transition from normal schooling to an online learning environment. In particular, keeping students at home dramatically influenced their level of engagement with content and their understanding of new knowledge (Yang *et al.*, 2020). In recent decades, student engagement has been extensively discussed in the literature as a key contribution to mathematics achievement and overall academic success (Wang *et al.*, 2016). Recent publications from the National Council of Teachers of Mathematics (NCTM) continue their advocacy for a vision of a mathematics classroom that focuses on positive engagement, including a mathematically rich classroom environment and classroom discourse, by encouraging students' active collaboration, connections, reasoning, sense-making, and problem solving (NCTM, 2009). While technology is used extensively nowadays in mathematics and all other subjects, engaging students in mathematics lessons has become its own challenge. Using digital technologies should not become a distraction that reduces students' engagement and procedural understanding of mathematical topics. Giving students a pedagogical, student-centered approach that provides an engaged learning environment in the age of unlimited educational and technological resources is an important challenge.

Flipped learning is a popular instructional innovation that relies on delivering technology, and it shows great promise in increasing mathematical achievement (Katsa *et al.*, 2016; Lo and Hew, 2017), supporting student teamwork and collaboration (Clark, 2015), and increasing higher-order thinking through problem solving (Gough *et al.*, 2017), especially in the K–12 paradigm in which students need more learning support. This study explores the learning of irrational numbers via a flipped learning instructional design. Making sense of irrational numbers is key to eighth graders' understanding of advanced mathematics (Yilmaz and Ay, 2018). The current study contributes to the literature by exploring the improvement in middle school students' performance and engagement in learning critical mathematics concepts, such as irrational numbers, through implementing flipped instructional innovation.

2. Literature Review

2.1. Student Engagement:

In mathematics education, students who engage with content are conscientious and willing to spend time on both in-class and out-of-class mathematics learning. Beyond this, they are eager to follow the teacher's instructions. They work diligently on problems, either with a surface-level or deep approach, and they spend their spare time learning (Kong *et al.*, 2003).

Research and educational organizations endeavor to define the complexity of the engagement construct. One example of such a definition is the National Survey of Student Engagement's (NSSE) characterization of student engagement at the college level. The NSSE provides several indicators that define student engagement as an academic challenge for higher education. These indicators include higher-order learning, reflective and integrative learning, multiple learning strategies, quantitative reasoning, collaborative learning, and discussions with diverse others. The Australasian Survey of Student Engagement (AUSSE) conceptualizes the construct of student engagement to measure students' academic challenges, active learning, social interactions, educational experiences, supportive learning environments, and work-integrated learning. Almutairi and White (2018) identified key factors and indicators of student engagement in the context of blended online education courses. These indicators are as follows: reflective and integrity learning, higher-order thinking, learning strategies, collaborative learning, and student–staff interaction.

For this current experiment, student engagement is defined as a multidimensional construct comprised of cognitive, behavioral, emotional, and social components (Fredricks and McColskey, 2012; Wang *et al.*, 2016). Behavioral engagement is defined in terms of involvement in academic and in-class activities, the presence of a positive manner, and the absence of disorderly behavior. It is related to positive school behaviors, such as following the school's rules and

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the absence of disruptive activities. Emotional engagement focuses on the extent of positive (and negative) reactions to teachers, classmates, academics, and the school. Cognitive engagement outlines a self-regulated theory about students' level of investment in learning. It includes being thoughtful, strategic, and willing to put in the effort to understand complex ideas or master difficult skills (Fredricks and McColskey, 2012; Wang *et al.*,2016).

2.2. Conceptual Framework of the Flipped Classroom:

Flipped learning is a teaching and learning approach that endeavors to give students ownership of their learning trajectories. It is defined as a pedagogical approach in which direct instruction moves from the group learning space to the individual learning space, and the resulting group space is transformed into a dynamic, interactive learning environment where the educator guides students as they apply concepts and engage creatively in the subject matter (Flipped Learning Network [FLN], 2014).

In a flipped mathematics classroom, curriculum materials are shared with the class via a technology-enhanced environment using video resources, which is followed by in-class mini-lectures, independent and group problem-solving activities, and integrated assessments (Lo *et al.*, 2017). In other words, students are exposed to the topic the day before class and engage in a low-level thinking process (i.e., remembering and understanding) while teachers spend their in-classroom time engaging students in high-level thinking processes with the content by involving them in creating, evaluating, and analyzing thinking processes about the particular mathematics concept. Much literature has discussed guidelines, design criteria, and conceptual frameworks of the flipped learning environment. The following sections review the literature on the essential flipped classroom guidelines and the elements adopted and implemented in this current study. As indicated in the literature, the four design guidelines are considered to be the most appropriate for implementation in mathematics classrooms in middle school education.

2.2.1. Focusing on Student-Centered Instruction

Through middle school mathematics, students must develop conceptual understandings of newly introduced concepts (e.g., irrational numbers) while building their confidence in strategically choosing and implementing procedures to solve a variety of complex problems, such as those involving linear equations of two variables and the Pythagorean theorem (National Governors Association, 2010). One of the main guidelines for a successful flipped lesson is emphasizing students' learning processes (Bergman and Sams, 2014). Classroom time is free from in-class direct instruction and allows for more student-centered learning activities, such as collaborative problem solving, constructive feedback, and discussion. This supports low-achieving students and improves their learning habits and communication skills (Lo and Chen, 2017; Akçayır and Akçayır, 2018). Teachers become more than simply presenters of knowledge and are more understanding of students' difficulties and their knowledge bases.

Using student-centered approaches in mathematics is at the core of effective mathematics teaching and learning practices. Students have the opportunity to construct their understanding of mathematical concepts by engaging in critical, in-depth, high-demanding thinking processes, such as reasoning, verifying, interpreting, connecting, communicating ideas, investigating or solving a variety of challenging and complicated problems (NCTM, 2000). Linda Gojak, a former NCTM president, indicated that teaching might succeed through the flipped model; however, teachers must maintain elements of student-centered, effective practices that help them engage with the mathematical content, create an environment conducive to learning, ensure access for all students, use questioning to monitor and promote understanding, and help students make sense of the mathematical content (Gojak, 2012). Since flipped models rely on technological factors to support a student's out-of-class learning process, Wang *et al.* (2010) consider that the inclusion and development of technology can help in the transition from teacher- to student-centered classrooms. A flipped classroom gives teachers the time and flexibility required to create the crucial student-centered learning environment by allowing for students' exploration of new concepts in their own time.

2.2.2. Continuous Environment Enhancement/Flexible Environment

In a flipped classroom, the teacher role is replaced, and direct instruction is altered. Flipping the mathematics classroom requires a distinct arrangement of the classroom environment (FLN, 2014; Bergman and Sams, 2014). The learning environment in this regard requires the practice of differentiated instruction through group and independent activities, the use of technological resources, such as videos and mathematics apps, in-class and online quizzes, extensive in-class group work activities, and high-ordered problem-solving activities (Lage *et al.*, 2000; Lo *et al.*, 2017; Lo and Hew, 2017; Muir, 2017). Flipped learning teachers are more flexible in managing learning resources to enhance students' conceptual understandings while substituting direct instruction. In their study of an economics college-level class, Lage *et al.* (2000) found that students indicated that they preferred to take flipped lessons. Furthermore, the students showed a positive attitude toward teaching because it was modified to their learning style and engaged all types of learners, such as experimental learners, female learners, collaborative students, independent learners, and direct learners.

In mathematics instruction, the learning environment can be structured to support students' conceptual understandings of mathematics. By reviewing 61 studies on flipped mathematics classrooms, Lo *et al.* (2017) reported a set of before-class, out-of-class, and in-class environmental design elements that led to effective mathematics learning. First, it is necessary to manage the before-class transition with students to introduce them to the new teaching method and allow them to understand the basics of applying this instructional design. Subsequently, students are requested to watch out-of-class videos and instructional materials. In-class environmental design elements and learning experiences are critical in flipped learning. First, teachers should pre-assess students' mathematical understandings of the new topic; then, they should present a mini-lecture based on the students' assessments, implement various real-world word problems, address students' questions, give them constructive feedback, and implement peer-assisted, small group-learning activities.

2.2.3. Collaborative Culture and Emotional Support

"Flipped learning is an inherently collaborative endeavor!" (Bergmann and Sams, 2014, p. 10). Collaboration in flipped design is constructed at all levels: students to students, students to teachers, teachers to teachers, and teachers to school administrators. Clark (2015) tested the impact of the flipped model on high school students' mathematics performance and engagement and found that the model improved students' communication and interaction skills with other students and teachers. The students enjoyed how the model allowed them to work collaboratively and cooperatively with their peers to complete tasks and projects. Additionally, they learned more effectively and improved their mathematics performance.

A study involving 25 first-year engineering students found that the flipped model provides professors with free time for communication with students in the class and for engaging students through individual discussions, particularly gifted students. Students in a flipped classroom display better individualization in terms of learning and have increased interest in cooperative learning, more than those in a traditional classroom (Voronina *et al.*, 2017). Providing students with communication platforms and learning management systems is an essential element in the flipped design, which emotionally supports students, manages their learning process, engages them in communication about the content, supports collaborative learning, and retains teacher–student learning discourse before and after the lessons (Lo and Hew, 2017; Clark, 2015).

2.2.4. Preserving Deep Learning and Creative Thinking Through High-Ordered Problem Solving

Beyond the focus of performing memorized procedures and routine algorithms, it is fundamental in a mathematics classroom to maintain free classroom time to involve students in meaningful activities, which increase the number of cognitively demanding mathematics tasks being implemented during classroom time (Stein *et al.*, 2009). The time spent on and the quality of mathematical work being implemented during class time in a flipped model matter the most in increasing students' conceptual understanding and engagement (Moore *et al.*, 2014). Flipped models have the potential to promote the enrichment of classroom time with cognitively demanding problem-solving activities. Lo *et al.* (2017) reviewed the literature that highlights the power of using flipped learning as a substantial instructional design tool to empower deep learning through solving real-world problems and demanding tasks. There are many advantages to the flipped classroom model that can provide mathematics instruction with free classroom time for rich activities that stimulate higher-order thinking through real-world problem solving (Kim *et al.*, 2014; Lai and Hwang, 2016; Moore *et al.*, 2014; Lo and Hew, 2017).

2.2.5. Student Engagement in the Flipped Classroom Model

The core idea behind flipping mathematics instruction is to increase students' engagement and opportunities to learn the content by moving the content from teacher–student-directed learning to students learning the content on their own via technology. High-level tasks and challenging homework problems are moved to classroom activities (Bergmann and Sams, 2014). The impact of flipped learning on a student's engagement level is a substantial and thought-provoking area explored extensively in recent literature (Clark, 2015; Fisher *et al.*, 2018; Lo and Hew, 2020; Bond, 2020).

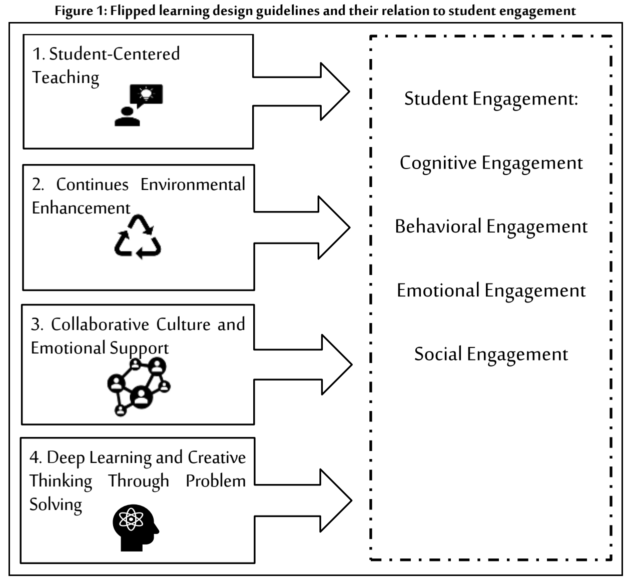
In a study that investigated the impact of the flipped model on high school students' engagement and mathematics performance, Clark (2015) disclosed the link between flipping the mathematics classroom and an increase in students' cognitive, social, and behavioral engagement. In a review of 107 papers, Bond (2020) demonstrated the impact of the flipped model on students' engagement, revealing that the instructional model greatly improved students' overall engagement, attitude, motivation, and self-efficacy and improved students' overall grades and performance. However, implementing instructional innovation brings many challenges and complexities. Some research still does not show a positive correlation between flipped learning and academic performance and engagement, while other studies have proven this positive relation (Lo and Hew, 2017).

The complexity and abstraction of many mathematics concepts and students' maturity and level of self-regulation may mediate this positive relationship. Thus, a gap exists among experimental flipped model research studies that seek to comprehend the nature of mathematical concepts that can be successfully delivered through the flipped model and maintain positive engagement and performance. Age-level boundaries and the design elements and experimental frameworks that may lead to positive results in a flipped classroom are essential areas of investigation. Thus, this current study examines the effects of a flipped classroom model on students' engagement and achievement, focusing on irrational numbers not used in any similar research.

Considering the previous literature, and as summarized through the study framework presented in Figure 1, this study seeks to answer the following research questions:

- Does the flipped learning environment positively influence eighth-grade students' engagement in an online mathematics classroom? What is the difference between students who learn using flipped models and those who do not in terms of their engagement level?
- Does the flipped learning environment positively influence eighth-grade students' achievements in mathematics?

Figure 1: Flipped learning design guidelines and their relation to student engagement



3. Methodology

3.1. Context, Participants, and the Experiment:

This study was conducted using a quasi-experimental design. During the fall semester of 2020, 60 eighth-grade students from two classes in a city in Saudi Arabia's Eastern Province were randomly identified and invited to participate. The convenience sample's ages ranged from 13 to 14 years. The students' families were informed about the research process and consented to participation.

A quasi-experimental design was employed to replace the design of real experiments when a random sampling selection process could not be done by the researchers and focused on interventions in real-world settings (Warner, 2008). One class was randomly identified as the experimental group and was taught using flipped learning methods by the mathematics teacher; the other class was the control group, which was taught the same lessons using the same mathematics teacher, who used her regular online teaching method. The mathematics teacher had a master's degree in mathematics, a higher diploma in education, and is well trained in integrating technology into mathematics classrooms.

This experiment required four weeks to complete the concepts in the chapter on irrational numbers. In keeping up with the Common Core State Standards and the Saudi mathematics curricula, the topic of irrational numbers is at an eighth-grade level, and both standards present similar topics (National Governors Association, 2010). Table 1 outlines the topics in this chapter.

Before the intervention, the mathematics teacher and the researcher collaboratively planned the flipped learning model using this study's four design guidelines:

- Ensure that teaching is always student-centered.
- Allow for environmental enhancement and modification throughout

the time of the experiment.

- Ensure an online collaborative culture and provide emotional support to the experimental group while applying the flipped model.
- Preserve deep learning and creative thinking through higher-order problem solving.

These four elements were introduced and discussed, and instructional events were planned by the teacher and the researcher to ensure the full implementation of the flipped learning design guidelines. The flipped instructional events, in light of the four guidelines, were divided into two stages: out-of-class events and in-class activities. For the out-of-class events, the mathematics teacher ensured that the experimental group engaged in direct instruction of the new mathematics concepts via instructional online videos or teacher-prepared videos, read the lesson from the textbook, solved online procedural homework problems via the school's learning management system (LMS), interacted with the teacher via a student-friendly social media platform, and used a graphing calculator, among other virtual mathematics apps, when needed. For the in-class activities, the mathematics teacher ensured that the experimental group engaged in pop quizzes about the newly learned lesson, received mini-lectures about the new concept, and engaged in highly demanding independent and group activities and real-world problem solving. An example of the implemented instructional events during the in-class and out-of-class periods used for teaching the chapter on irrational numbers to the eighth-grade experimental group students is described in Table 1.

Table 1: Implemented instructional flipped learning events for in-class and out-of-class activities

Topic	Lesson Objective	Online Homework Before Class	In-Class Activities
Introduction to irrational numbers	Define irrational numbers and locate them on a number line. Find and estimate square roots. Classify real numbers as rational and irrational.	Watch an online video; read the textbook. Solve online procedural homework problems. Answer the teachers' follow-up questions, and ask queries when needed via the LMS.	Mini-lecture on the new concepts Go over homework problems. Group activity: What is the radius of a sphere when the surface area is known?
Exploring the Pythagorean theorem	Explore the Pythagorean theorem. Discover the equation that relates the side length of a right-angled triangle.	Watch the introductory video. Draw the assembly of the Pythagorean theorem on grid paper. Watch and interact with a virtual video on the meaning of the Pythagorean theorem. Answer teachers' follow-up questions, and ask queries when needed via LMS and other student-friendly social media.	Mini-lecture on the theorem. Discover the equation that relates the side lengths of a right-angled triangle.

3.2. Data Collection Instruments:

This study used the Mathematics and Science Engagement Scale (MES), which was initially created to measure student engagement among middle school and high school students in STEM subjects (Wang *et al.*, 2016). In its original form, the scale consisted of 33 items divided into four subfactors: cognitive engagement, behavioral engagement, emotional engagement, and social engagement.

The original reliability of the scale was calculated by Wang *et al.* (2016), and the Cronbach's alpha coefficient of reliability was 0.93. The total scale and coefficient for each subfactor were 0.75, 0.82, 0.89, and 0.74, respectively. For this current study, the Cronbach's alpha coefficient of reliability was measured at 0.911, which is acceptable, suggesting that the instrument was reliable for these specific participants.

The MES scale was employed in this study to investigate the impact of flipped learning on students' engagement levels. The scale used was a five-point Likert scale ranging from *very true* to *not at all true*. The items were self-reported, and the students measured their level of engagement in mathematics learning. Of the 33 items, 12 items were measured negatively, meaning that the point *very true* was scored as 1, while the point *not true at all* was scored as 5. The MES was administered equally to the experimental and control groups twice: once before the intervention and once at the end. We assumed that the eighth-grade students answered the items based on their reflections of their level of engagement with the content of irrational numbers.

This study also explored the impact of the flipped learning model on students' mathematical achievement and performance regarding irrational numbers. For this part of the investigation, mathematical achievement scores were obtained to investigate the students' performances. This kind of achievement test is regularly implemented by the school's administration at the end of each chapter and was prepared by the mathematics teachers who were teaching the same grade level, with the guidance of the district's mathematics supervisor. The highest score for the achievement test was 10. It measured the concepts taught in the chapter, focusing on conceptual understanding and procedural fluency. As indicated by the mathematics teacher of the eighth-grade class, a result of 8 or higher was considered excellent, 6 to 7.9 was considered very good, 3 to 5.9 was a fair result, and less than 3 was unacceptable.

The data from the achievement test were provided to the researcher at the end of the four-week intervention and reflected the students' level of understanding of the concepts taught in the irrational numbers chapter. The results of the achievement test by both the experimental and control groups were provided to explore the impact of administering the flipped learning model on students' mathematics performance.

4. Data Analysis and Findings

4.1. Pretest Results:

This study examined the impact of the flipped learning model on eighth-grade students' mathematics engagement and achievement regarding irrational numbers. To ensure balance and reliability at the outset of the study, the difference between the experimental and control groups in their level of engagement was calculated using an independent samples t-test, and it showed no significant results (see Table 2). Thus, based on the analysis of the MES pretest, it is believed that the two groups had equal levels of mathematics engagement. For the achievement level, neither group had been taught irrational numbers. Following Saudi Arabia's national mathematics curriculum, both groups were equal in their knowledge of irrational numbers. Therefore, the potential threat of initial variance among students' mathematics engagement levels and achievement could be excluded.

Table 2: Independent sample t-test for the two groups before the experiment (N = 60)

Group	M	SD	SE	*t*	df	Sig.
Pre-control	3.6537	0.49712	.09076	0.550	58	.584
Pre-experiment	3.7265	.52757	.09632			

4.1.1. The Impact of the Flipped Learning Model on Student Engagement

In this study, to measure the influence of the flipped learning model on the level of student engagement, the experimental group's members were asked to rate their responses to 33 items on a five-point Likert scale as follows: 1 = not true at all, 2 = usually not true of me, 3 = occasionally true of me, 4 = usually true, and 5 = very true. The MES helped to provide an understanding of the students' level of engagement after studying irrational numbers in the flipped learning environment and to rate their cognitive engagement, behavioral engagement, emotional engagement, and social engagement.

The results presented in Table 3 indicate the mean scores of the students in the experimental group when they rated their level of engagement using the MES. The table presents the percentage of students in the experimental group who rated themselves high on the MES (i.e., a 4 or 5 score) and the percentage of students who rated themselves low in the same experimental group (i.e., a 3, 2, or 1). The results of the mean of the MES items were categorized as very high (ranging above 4.5), high (ranging from 4.0 to 4.4), slightly high (ranging from 3.5 to 3.9), and medium (ranging from 3 to 3.4). This process helped with classifying and interpreting the overall level of engagement as reported by the students when they learned mathematics using the flipped model.

Table 3 also shows that, in most items, experimental students expressed a high to medium level of engagement when learning mathematics in the flipped environment ($0.3 < \text{mean} < 4.5$). A medium level of engagement was expressed by some students ($0.3 < \text{mean} < 3.4$), especially for items 2, 3, 6, 8, 13, 14, 26, and 27. The mean score for the cognitive engagement factor in the experimental group was 3.3, the behavioral engagement factor was 3.7, the emotional engagement factor was 3.9, and the social engagement factor was 3.8, indicating slightly high to medium mean scores among the four factors in the experimental group.

As shown in Table 3, the mean score for the cognitive factor was a medium mean of the students' levels of mathematics engagement. In the literature, as noted earlier, the construct of cognitive engagement demonstrates the level of students' willingness to put effort into learning mathematics. Moreover, 90% of the students who studied in the flipped learning model rated themselves high in checking their mathematics work for correctness, 86% rated themselves high in using multiple methods to solve problems, 69% practiced mathematics connections between concepts, and 80% reported themselves high in understanding their mistakes. Negatively worded items revealed that only 20% of the students focused on memorizing their answers when learning mathematics, 23% studied only the easy parts for exams, and only 13% shared that they learned mathematics only to pass the subject. The results for the cognitive engagement factor revealed a medium cognitive engagement level among most of the participating students due to learning mathematics through the flipped model. Nonetheless, the mean score for the cognitive engagement construct was lowest among the four MES factors.

The total mean scores for behavioral engagement and social engagement indicated slightly high levels of engagement among middle school students who participated in the flipped model of instruction. Therefore, 73% of the students identified themselves as high in items of the behavioral engagement construct concerned with concentration, effort, and repetition for learning. Additionally, 86%, 60%, 63%, and 83% of the participating students in the experimental group rated themselves as high on items related to completing work, communicating mathematics, class participation, and quicker understanding, respectively. Likewise, 86%, 76%, 56%, and 80% of the participating students in the same group rated themselves high on the items of the social engagement construct related to learning from others, understanding others' ideas, collaborative learning, and peer support, respectively. Negatively worded items, such as items related to rigidness, selfishness, and individualism, were rated high by a minuscule number of students, which indicated that the majority of the experimental group perceived themselves positively in most items in the social and behavioral engagement factors.

Table 3: Measuring students' mathematics engagement in flipped classrooms (N = 60)

Item	M	SD	Not true, usually not true, occasionally true (%)	Vey true/usually true of me (%)
Cognitive Engagement				
Checking for mathematics work correctness	3.966	1.12903	10%	90%
Multiple solutions for problem solving	3.2000	0.9965	13.4%	86.6%
Mathematics connection	3.466	1.3829	30.3%	69.7%
Understanding mistakes	3.9667	1.4015	20%	80%
Focusing on memorizing the answers vs. working hard	3.6667	1.4463	80%	20%
Determined to get the right answer	3.000	1.5974	40%	60%
Studying the easiest parts	3.5667	1.5974	76.7%	23.3%
Learning only to pass	1.6333	1.18855	83.3%	16.7%
Behavioral Engagement				
Concentration	3.8000	1.58441	26.7%	73.3%
Effort	3.7000	1.6220	26.7%	73.3%
Repetition	4.1000	1.4227	26.6%	73.4%
Completing homework	4.0667	1.3373	13.3%	86.7%
Mathematics communication	3.0667	1.6386	40%	60%
Class participation	3.000	1.3645	36.7%	63.3%
Class distraction	3.8333	1.5331	76.7%	23.3%
Understanding faster	4.1333	1.4558	16.6%	83.4%
Emotional Engagement				
Class motivation	3.7000	1.2359	13.3%	86.7%
Enjoying learning mathematics	4.0667	1.257	23.3%	76.7%
Enthusiasm	4.1667	1.2576	20%	80%
Excitement	3.6000	1.4527	16.7%	83.3%
Frustration	3.466	1.59164	66.7%	33.3%
Boredom	3.800	1.4239	73.4%	26.6%
Readiness	4.4333	1.2507	13.4%	86.6%
Carelessness	4.533	1.1665	90%	10%
Pessimism	4.1667	1.26173	86.6%	13.4%
Worry	3.4667	1.47936	70%	30%
Social Engagement				
Learning from others	3.4667	1.13664	13.4%	86.6%
Understanding other ideas	3.500	1.2798	23.3%	76.7%
Collaborative learning	3.8667	1.13664	43.3%	56.7%
Peer support	3.6000	1.49943	20%	80%
Rigidness	3.9000	1.49366	80%	20%
Selfishness in learning	4.1667	1.23409	86.7%	13.3%
Individualism	4.3000	1.46570	83.7%	16.3%

As shown in Table 4, the mean score (3.9) for the emotional engagement factor was the highest among the four factors. Students in the experimental group rated themselves as having a high level of engagement in items related to class motivation, enjoyment of learning mathematics, enthusiasm, excitement, and readiness. The data revealed that the flipped learning model has a high impact on students' emotional engagement and slightly less impact on students' cognitive engagement. Table 5 shows the results of the paired-sample t-test between the factors of the experimental group, which revealed significant results only when a factor was paired with cognitive engagement.

Table 4: Paired-samples t-test of two factors of the engagement scale within the experimental group (N = 30)

	Experimental Group					
	M	SD	SE	T	Df	Sig.
CE*BE	0.35417	0.5675	0.07327	4.834	59	.000*
CE*EE	0.47042	0.79054	.10206	4.609	59	.000*
CE*SE	0.36637	0.72103	.09308	3.936	59	.000*
BE*EE	0.11625	0.63764	.08232	1.412	59	0.163
BE*SE	.01220	0.51731	.06678	0.183	59	0.856
EE*SE	.10405	0.66412	.08574	1.214	59	0.230

4.1.2. Differences Between the Experimental and Control Groups

This current study explored the following question: What is the difference between students who learn using flipped models and those who do not in terms of their engagement level? An independent sample t-test was used to test the following hypothesis: Statistically significant differences exist between the means of the control group and the traditional group at the 0.05 level. In other words, an independent samples t-test was performed to assess whether the mean of students' levels of mathematics engagement differed significantly between the group of students who were taught irrational numbers using the flipped learning model and the control group.

The results presented in Table 5 demonstrate no significant difference ($p > 0.05$) between the mean of the flipped group (mean = 3.69, SD = 0.74) and the control group (mean = 3.52, SD = 0.52) in their level of mathematics engagement. This comparison was made given the results presented in Table 2, in which both groups were equal in their level of engagement at the beginning of the intervention.

Table 5: Independent sample t-test of the two groups (N = 60)

Group	M	SD	SE	*T*	Df	Sig.
Post-Control	3.5273	0.58997	0.10771	0.985	58	.331
Post-Experiment	3.6974	.74545	.13610			

Additionally, based on the data presented in Table 6, there was no significant difference between the flipped group and the control group in the four dimensions of the MES. Although the mean score of the experimental group was slightly higher than that of the control group in terms of the level of engagement, this higher mean was not significant.

Table 6: Independent samples t-test of the four factors of the engagement scale (N = 60)

	Post-Control Group			Post-Experimental Group					
	M	SD	SE	M	SD	SE	T	df	Sig
CE	3.3208	0.51767	0.09451	3.3083	0.64053	0.11694	0.083	58	0.934
BE	3.6250	0.63058	0.11513	3.7125	0.81747	0.14925	0.464	58	0.644
EE	3.6300	0.92742	0.16932	3.9400	1.0344	0.1888	1.222	58	0.227
SE	3.5333	0.71766	0.13103	3.8286	0.83670	0.15276	1.467	58	0.148

* P < 0.05

4.1.3. Results Derived from the Mathematical Achievement Score

The results of the statistical analysis presented in Table 7 reveal the differences between the experimental and control groups in terms of their achievement scores, which measured their performance and understanding of concepts in the chapter on irrational numbers. The mean scores for both the experimental and the control groups had excellent results (mean > 6). However, our results revealed no significant difference between the two groups of students (p > 0.05), with a slightly higher mean score in the experimental group.

Table 7: Independent sample t-test of students' achievement scores (N = 60)

Group	M	SD	SE	*t*	Df	Sig
Control	6.4697	1.92286	0.35106	0.441	58	0.661
Experimental	6.7027	2.16614	0.39548			

5. Discussion and Implications

During the challenging online schooling period, students have been challenged to learn mathematics topics and maintain a level of cognitive, behavioral, emotional, and social engagement in the online environment. This study highlighted the role of the flipped learning model and its effectiveness in students' mathematics engagement and learning.

The model was tested on eighth-grade students in Saudi Arabia through the teaching of irrational numbers using the flipped model. Guidelines to support the successful implementation of the model, as per the current literature, were emphasized. The results of the data analysis demonstrated an overall medium to a slightly high level of engagement and mathematics performance among the experimental group. A difference between the experimental and control groups in terms of mathematics engagement levels and achievement scores was not found in this study. Nonetheless, this study acknowledges that the study's duration and the limited number of participants are considered limitation factors that may interfere with the generalizability of the study.

As noted by Lo and Hew (2017), this result is acceptable and predictable in quasi-experimental designs involving the flipping of model environments for several reasons. First, the experiment's duration was very short (four weeks). Second, the complexity of the topic may have meant that the mathematics teacher put in the same effort with both groups. Third, the implementation of technology in the flipped classroom may have prevented students from making extensive progress in their level of engagement, in contrast to the control group. It was found that the mean overall engagement score of the experimental group was acceptable. In sum, the novel implementation of the flipped learning model affected the experimental group positively in terms of engagement level and offered them support, leading to excellent results on the achievement test.

Through this experiment, the level of engagement in the cognitive dimension was measured as medium. Flipped learning provides support for healthy cognitive mathematical practices, such as practicing mathematical connections, improving hard work beyond memorization, and providing a productive environment for multiple solutions because of differentiated instruction (Moore *et al.*, 2014; Willey and Gardner, 2014; Lo and Hew, 2017). Future studies are needed to analyze homework and group work to explore the level of students' improvement in the cognitive lens and to better understand the level of students' creativity and high-ordered thinking. Prospective studies can investigate the differences between multiple flipped groups with different types of mathematics tasks, different levels of student collaboration, and multiple cognitive levels of mathematical tasks.

In the study data, the level of engagement in the behavioral domain measured slightly higher than the other types of engagement. A large proportion of the experimental group reported themselves as high in items related to concentration, mathematics communication, and class participation after they were taught via the flipped model. From the beginning of this study, the model design focused on collaboration between students through group activities and open discussions with the mathematics teacher through online systems and social media.

Mathematics instruction with flipped learning can be enriched through the process of communicating mathematics content. Similar results from other studies show that the activities of behavioral engagement, such as communication, participation, and classwork completion, improved because of the flipped model (Hodgson *et al.*, 2017). The act of classroom communication and positive behavioral activities have their benefits; thus, the flipped model in this study may have provided an environment for flourishing classroom communication. Although this result was not obtained through real sitting observation in flipped and non-flipped classrooms, the flipped model, in its ideal environment, allows for in-class communication and participation because it frees classroom time from direct instruction. Here, teachers can use this instructional space to support students' communication, participation, and mathematics-related discourse. Therefore, the flipped model provides a learning environment that supports classroom communication (Bergmann and Sams, 2014; Clark, 2015).

The flipped learning model provides emotional support for students during the process of learning mathematics. This study's results demonstrate that emotional items scored highest among the means of the four MES subfactors. These findings are thought provoking. They indicate promising outcomes for items related to enjoying mathematics, enthusiasm, and readiness. As stated in Wang *et al.*'s (2016) research, students who practice positive emotional engagement are those who enjoy learning, value social competence with peers, show positive interest and belief in mathematics, and are likelier to attain high academic achievements. One of the main implications of this current study is that further research should expand the exploration of the emotional construct revealed when learning mathematics in a flipped environment and characterize key classroom practices that can empower students' emotional patterns.

Irrational numbers are a concept that is considered challenging for eighth-grade students, as they are a transition to the more abstract mathematics covered in later grades. In response to various mathematics reform agendas, mathematics discourse and classroom communication receive much attention as tools to help students master critical mathematical concepts. Thus, grounding social engagement and relating it to the flipped model is essential.

Classroom time can be used for teachers to pose critical questions, support group thinking, present various solutions, and clarify common group and individual mistakes in learning mathematics. This study indicates encouraging results regarding the social engagement domain from the participants' ratings of items related to social learning activities, such as learning from others, understanding others, and collaborative learning. However, a close investigation is needed to code and interpret students' social interactions and levels of learning from their peers and to study whether this social engagement can improve or whether it contradicts mathematics performance and cognitive engagement. Cultural factors were not of interest in this study; thus, classroom social interactions influenced by cultural aspects in the flipped environment may be a poignant area of discovery.

While this study reports promising results regarding engagement and performance within flipped mathematics instruction, the results for the flipped learning environment are not significant. Challenges and obstacles regarding the flipped learning approach in relation to learning mathematics have been addressed in the literature.

Challenges perceived by teachers and students include the novelty of experiences, which would require further training, competencies, workload, time-consuming activities, anxiety, and resistance to change. Positive results cannot rely solely on implementing the flipped model but may be influenced by other factors, such as parents teaching new materials or assistance from peers or the internet.

Another disadvantage of the flipped model is its reliance on the level of accessibility to technology. Students with access to technology and everyday interactions with games, YouTube channels, and phone applications are expected to learn better in a technological environment than students with limited access to technology.

Another challenge that may be addressed is the nature of learning mathematics, which requires collaboration, face-to-face interaction, and rich classroom discussion about concepts with instant feedback. These factors are counted as obstacles to the flipped learning model, which has been discussed in many studies (Al-Abdullatif, 2020; Lo *et al.*, 2017).

6. Conclusion

Our innovative, technology-enhanced world of education is pushing itself to search for, adopt, and investigate instructional designs that are "technology friendly." This study sought to improve students' learning of irrational numbers and their level of engagement through the enactment of technologically enhanced flipped learning environments. This study proposes that students in flipped learning environments engage cognitively, behaviorally, emotionally, and socially. Regarding our specific topic, the students performed well on standardized testing of irrational numbers. While the comparison between the flipped and control groups was not significant, this study provided valuable results for teaching abstract and complex mathematics concepts via a novel flipped learning instructional design.

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