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Model Order Reduction by Using Improved Approximation Techniques

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ABSTRACT

A simplified approach for model order reduction (MOR) is presented in this article using the balanced singular perturbation approximation (BSPA) approach applicable to large-scale linear dynamical (LSLD) systems. The reduced system was so designed to preserve complete parameters of the original system with reasonable accuracy, employing MOR. The approach is based on the retention of the dominant states of the system and comparatively less important ones. The reduced system comes from the preservation of the dominant states (say 'desirable states') of the original system and, thus, from stability to preservation. The key disadvantage of the Balanced Truncation approach is that the ROM steady-state values do not correspond with the higher-order systems. This drawback has been eliminated in the proposed approach, which leads to hybridisation of balanced truncation and singular perturbation approximation into a novel reduction method without the loss of retaining its dynamic behaviour. The proposed approach has been tested on LSLD systems and the results obtained show the efficacy of the approach. The methodology presented has been tested on two typical numerical examples taken from the literature review to examine the performance, precision and comparison with other available standard order reduction methods.

1. Introduction

The most important problem in an appearance on the complex activities of the higher dimension system is that it occurs in many areas, including complicated transport, ecological systems, electrical power equipment, aeronautics and hydraulics (Suman *et al.*, 2020a; Sambariya *et al.*, 2016; Sikander *et al.*, 2015b; Antoulas *et al.*, 2000).

All these complex and large systems with conventional techniques are difficult to model. The combination of these is also considered to be big (large) if it wishes to be detached for each numerical measurement to many structured machinery or small structures for practical purposes (Suman *et al.*, 2020b; Mohamed, 2018; Boley and Datta, 1997). Then, perhaps a system is complex and wide enough to fail to generate the proper solutions with realistic computational efforts by conventional modelling, analysis, device design and approximation strategies (Suman *et al.*, 2019; Willcox *et al.*, 2002). Studying this physical system (Schilders, 2008) starts with structuring the model, which can be considered as an enthusiastic example of this kind of structure, which is motivated by a task of control in preparing and evaluating a model (Suman *et al.*, 2019). We are presenting a high stage of negotiation on computing in this first segment, which is important for detailed incident model observations in perspective and industry implementation (Mohamed, 2018; Gugercin and Antoulas, 2006).

Several MOR solutions were mainly provided in two ways, namely frequency and time domain (Sandbergr *et al.*, 2004). Researchers' reduction techniques have both benefits and inconveniences. One common weakness in the methods is that even if the HOS is stable, the reduced-order system is unstable (Suman *et al.*, 2019; Gupta *et al.*, 2018; Sikander *et al.*, 2015b) .The other drawbacks are the low precision in average ranges as well as high frequency and the non-minimum phase characteristics (Cao *et al.*, 2019;Benner *et al.*, 2015). Based upon the dominant poles method, numerous mixed methods have been suggested by Singh *et al.* (2016). The continued method and time matching fraction expansion can produce stable systems models. In the literature search, there are numerous approaches for reducing models of higher-order dynamic systems, such as a the reduced-order model (ROM) algorithm, which was presented with a Pade approximation (Parmar *et al.*, 2007; Mittal *et al.*, 2004; Mukherjee *et al.*, 1987; Shamash, 1974). Moore (1981) initially suggested a model order reduction of the state linear time-invariant system based on the theory of balanced realisation, in which the realisation term balanced is selected for the system state configuration and partitions of the modes (Fernando and Nicholson, 1983a). The BT, reduced system obtained through a balanced realized model subsequently elimination of less controllable and less observable states. It has been found that the model so obtained does not retain the steady-state and DC gain of the LSLD system. Vincenza *et al.* (1982) noted that a weak subsystem removed can be used to preserve the steady-state gain of the balanced truncation using a singular perturbation approximation approach (Huang, *et al.*, 2013; Benner *et al.*, 2010; Škatarić *et al.*, 2010; Clapperton *et al.*, 1996; Samar *et al.*, 1995; Liu *et al.*, 1989; Al-Saggaf and Franklin, 1988; Glover, 1984). Preserving the ratio of the steady-state output to the steady-state input (DC gain) of the balanced truncation model for the minimal system using a singular perturbation approach can be used to reduce the system to stable, minimum and internal balancing (Liu and Anderson, 1989b).

In this paper, a hybrid approach with BT and SPA Approach applicable to the higher-order system with excellent DC gain matching was proposed. The advantage of the approach lies not only to the matching of steady-state but its applicability to the large-scale linear dynamical system as well, which has been confirmed over some test systems taken from published work to validate the efficacy of the proposed approach.

The reduced-order approximation estimate for linear dynamic systems is approximated. Consider taking a -LTI SISO system of continuous-time as defined by

$$\Sigma: \underbrace{\begin{cases} \frac{dx}{dt} = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases}}_{(n)\ high-dimensional\ system} \Leftrightarrow \Sigma := \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right) \tag{1}$$

where $x \in \mathbb{R}^n$ is n-dimensional of the system state(1), $y \in \mathbb{R}^p$ is the system output of $\sum$, u(t) is the manage input, A $\in \mathbb{R}^{n\times n}$, B $\in \mathbb{R}^{n\times m}$, C $\in \mathbb{R}^{p\times n}$ and $D \in \mathbb{R}^{p\times m}$. The systems are stable and minimal in all states. $x \in \mathbb{R}^n$ n is state variables of

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higher order system equation (1) that it is a high-dimensional system, with n ranging from a few $k \rightarrow n$ to thousands (n) as in control complex activities of the higher dimension system (Antoulas, 2005; Zhou et al., 1999; Boley and Datta, 1997; Moore, 1981a).

Consider the n-dimensional dynamic system transfer functions of SISO (1) (Sikander *et al.*, 2015a). Written in the form of transfer matrix is defined as

$$G(s) = C[sI_n - A]^{-1} B + D \tag{2}$$

$$G(s) = \frac{\sum_{i=0}^{n-1} n_i s^i}{\sum_{i=0}^{n} d_i s^i} \tag{3}$$

Where n^th order higher order is represented by $G(s)$. And, also, n_i, d_i the known scalar coefficients parameters by the creative higher-order system and reduced-order Model.

The higher-order system is to reduce to LSLD, and ROM can be written as follows

$$\underbrace{\Sigma_r : \begin{cases} \frac{dx_r}{dt}(t) = A_r x_r(t) + B_r + u_r(t) \\ y_r(t) = C_r x_r(t) + D_r u_r(t) \end{cases}}_{(r)\ low-dimensional\ system} \Leftrightarrow \Sigma_r := \left(\begin{array}{c|c} A_r & B_r \\ \hline C_r & D_r \end{array} \right) \tag{4}$$

Where r is always less than n, so, to achieve the desired transfer function of HOS approaches and the reduction approaches to model typically diverge in the estimated error, which is reduced. The subscript 'r' is used during this brief to denote ROM-related parameters. The main aim of this role is to calculate the parameters of r^{th} -order r always less than n, which is the order of the higher-order system, the equation (6).

The desired reduced-order model $G_r(s) = R(s)$ has the following transfer matrix:

$$R(s) = C_r (sI_{n_r} - A_r)^{-1} B_r + D_r \tag{5}$$

$$R(s) = \frac{\sum_{j=0}^{r-1} m_j s^j}{\sum_{j=0}^{r} n_j s^j} \tag{6}$$

Where $R(s)$ r^{th} reduced order of higher order. And also m_j, n_j the known scalar coefficients parameters of reduced-order model (Prajapati *et al.*, 2019c).

2. Balanced Singular Perturbation Approximation

The reduction method for higher-order systems is explored here via the balanced singular perturbation approximation (BSPA). It is hybridization of BT and SPA. In the BT method, all balanced systems are separated into two parts as a slow and fast mode by defining the lower Hankel singular values (HSV) as fast mode, with the others defined as a slow mode. First, it is possible to obtain a ROM by setting the derivative of all fast mode states equal to zero. The steps of order reduction using BSPA approach are given below:

Step 1: Let us consider a linear, time-invariable and asymptotically stable G(s) system with minimal realisation.

The controllability grammian (CG) and the grammian observability (OG) of the system are as follows:

It is well known that these grammians satisfy the following Lyapunov equations (9) and (10) to found the controllability grammian (C_g) and observability grammian (O_g):

$$C_g = \int_0^\infty e^{A\tau} BB^T e^{A^T\tau} d\tau \tag{7}$$

$$O_g = \int_0^\infty e^{A^T\tau} C^T C e^{A\tau} d\tau \tag{8}$$

$$AC_g + C_g A^T + BB^T = 0 \tag{9}$$

$$A^T O_g + O_g A + CC^T = 0 \tag{10}$$

A realisation (A, B, C, D) of the system $G(s)$ is said to be internally balanced if $C_g = O_g = \Sigma = Diagonal\{\sigma_1, \sigma_2, \sigma_3 \cdots \sigma_n\}$ where Σ is a diagonal (and indefinite) matrix if such a realisation exists, and the respective system energy is indicated here. These singular values are normally ordered for convenience to truncate the state corresponding to smaller Hankel singular values as $\sigma_i \geq \sigma_{i+1}, i = 1,2,...n-1.$ (López-Caamal and Marquez-Lago, 2014; Segalman, 2007; Gugercin *et al.*, 2004; Marsden *et al.*, 2002; Zhou *et al.*, 1999; Boley and Datta, 1997; Moore, 1981a).

2.1. Assumption:

The nth-order dimensional system is an asymptotically stable system and also minimal. Moreover, the state-space system equation (1) or the pair (A, B) state is controllable if, and only if, the n × nm state controllability matrix and pair (A, C) are observable in the np × n observability matrix (Moore, 1981).

According to the above statement, both grammians are a positive definite and unique symmetric matrix explanation to the couple of grammians since their implementation is minimal (Imran *et al.*, 2014; Gugercin *et al.*, 2004; Zhou *et al.*, 1999).

Step 2: Determination of Hankel singular values (HSV): Obtain Hankel singular values (HSVs) of the system by finding the square root of the eigenvalues of the product of OG and OG.

$$HSV = \sqrt{\lambda_i (C_g O_g)} = [\sigma_1, \sigma_2, \sigma_3 \cdots \sigma_r, \sigma_{r+1} \cdots, \sigma_n] \tag{11}$$

Where σ is called the Hankel singular value of the nth mode and λ_i is the real diagonal eigenvalue of the matrix.

It provides a measure of most controllable/observable state. Compute the Hankel singular values (HSV) of the dynamic system. The singular Hankel values measure each state's contribution to the input/output behaviour in state coordinates that equalise the input-to-state and output energy transfers. In addition, it indicates the desired and optimal order reduction of the original system. It is also the square root of the eigenvalues of controllability grammian and observability grammian ($C_g O_g$) (Rowley, 2005; Antoulas, 2005; Sikander *et al.*, 2017).

The Cholesky factor (CF) problem of the positive–definite symmetric is frequently initiated using CF factorisation (Boley and Datta, 1997; Pernebo *et al.*, 1982). The lower triangular matrices (CF) Q_c and Q_O of controllability grammian C_g and observability grammian O_g are determined as (López *et al.*, 2014; Boley and Datta, 1997; Al-Saggaf and Franklin, 1988).

$$Q_c = C_g C_g^T \tag{12}$$

$$Q_o = O_g O_g^T \tag{13}$$

The singular value decomposition (SVD) $Q_o^T Q_c$ is obtained as follows (Boley and Datta, 1997; Liu *et al.*, 1989; Enns, D.F., 1984)

$$SVD\left(Q_o^T Q_c\right) = U \Sigma V^* \quad (14)$$

Where, U and V are right and left vectors, known as orthogonal columns matrix. By using a non-singular matrix W (transformation) the model can be transformed into a balancing model with help of transformation matrix, which can be attained as follows (Imran *et al.*, 2014; Gugercin, 2008; Zhou *et al.*, 1999; Boley and Datta, 1997).

$$W = Q_c V \Sigma^{-\frac{1}{2}} \quad (15)$$

$$\Sigma = Diagonal\{\sigma_1, \sigma_2, \sigma_3 \cdots \sigma_n\} \quad (16)$$

Then the system with coefficient matrix

(WAW^{-1}, WB, CW^{-1})

Where W is a non-singular Similarity transformation matrix.

In brief, the following transformation obtained a reduced order model to balanced system.

$$G_{Bal}(s) = \left[\begin{array}{c|c} WAW^{-1} & WB \\ \hline CW^{-1} & D \end{array}\right] \quad (17)$$

$$G_B(s) = \underbrace{\left[\begin{array}{c|c} A_B & B_B \\ \hline C_B & D_B \end{array}\right]}_{Balanced\ system} \quad (18)$$

At this point, we can partition the system's balanced realisation (A_B, B_B, C_B, D_B) and retain the larger singular values of the system that correspond to the strongly controllable and observable states of the system.

Now, partition the balanced realisation and the grammian $\sum$ conformal as

$$= \underbrace{\begin{bmatrix} A_{11} & B_1 \\ C_1 & D \end{bmatrix}}_{Strong-Subsyustem} + \underbrace{\begin{bmatrix} A_{22} & B_2 \\ C_2 & 0 \end{bmatrix}}_{Weak-Subsystem} \Leftrightarrow \Sigma : \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \quad (19)$$

Where A_{11} and $\sum_1$ are $r \times r (r\ less\ than\ n)$ matrices.

We call this reduced order model a balanced system approximation of direct-truncation (DT). There are some well-known results on the approximation (Liu and Anderson, 1989a; Moore, 1982).

2.2. Lemma (Pernebo and Silverman, 1982):

The subsystems (A_{ii}, B_i, C_i) $(i = 1, 2)$ are internally balanced with grammian $\Sigma_i (i = 1, 2)$. For proof refer to Liu *et al.* (1989) and Enns (1984).

2.3. Lemma (Pernebo and Silverman, 1982):

The matrices A_{ii} $(i = 1, 2)$ are asymptotically stable, i.e., $\mathrm{Re}(\lambda_k\{A_{ii}\}) < 0$, for all k $(i = 1, 2)$ if $\sum_1$ and $\sum_2$ have no diagonal entries in common. Further, the subsystem (A_{11}, B_1, C_1) is controllable and observable. For proof refer to Liu *et al.* (1989) and Enns (1984).

Where λ_k is eigenvalue and A_{ii} is a subsystem of the original system $(i = 1, 2)$.

Let us now focus upon using singular perturbation technique to reduce the order of a linear time-invariant system. (Fernando *et al.*, 1982). Equation (19) is accomplished by a model of strong and weak subsystems that have been minimally realised. Singular perturbation Approximation (SPA) can also be used for Equation subsystems (19).

In the model BT 'r' are kept balanced, strongly controlled and observed and even weakly controlled and/or observed. The weakly truncated state is used to (preserve) maintain the original system DC gain using SPA in the model (Kumar *et al.*, 2012; Safonov *et al.*, 1989). If the system is present in quick subsystems (Fernando and Nicholson, 1983), the transients associated with those subsystems will disappear quickly and, thus, the overall contribution to the system's impulse response is small. Therefore, fast subsystems are characterised by the relatively small diagonal elements of the weal matrix. The concerned researcher is referred to (Antoulas *et al.*, 2000; Saksena *et al.*, 1984; Kokotovic *et al.*, 1976) for overviews of the technique. With a balanced realisation defined as follows:

$$\left.\begin{array}{l} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}}_{A_B} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}}_{B_B} u \\ y = \underbrace{[C_1 \quad C_2]}_{C_B} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + D_B u \end{array}\right\} \quad (20)$$

Or

$$\underbrace{A_{11} \quad A_{12} \quad B_1}_{Slow\ Time} \qquad \underbrace{A_{21} \quad A_{22} \quad B_2}_{Fast\ Time} \quad (21)$$

These are fast and slow modes of SPA approaches (Gajic *et al.*, 2001; Fernando *et al.*, 1983b).

Because the system is balanced, states conforming to smaller Hankel singular values $\sum_2$ represent the fast dynamics of the systems. Based on the concept of the singular perturbation approach (Kumar *et al.*, 2012), we set the derivative of the approximate $\sum_2$ system for all states to zero.

By comparing the derivative of the weakly subsystem to zero below, the BSPA model can be achieved (Guiver, 2019; Kumar *et al.*, 2012; Liu *et al.*, 1989).

Now, after implementation of the above steps to partition the balanced system (A_B, B_B, C_B, D_B) conformally as in (22). BSPA is define as given by $G_{BSPA}(s) = G_r(s)$:

$$G_{BSPA}(s) = \underbrace{\left[\begin{array}{c|c} \hat{A} = A_{11} - A_{12}A_{22}^{-1}A_{21} & \hat{B} = B_1 - A_{12}A_{22}^{-1}B_2 \\ \hline \hat{C} = C_1 - C_2A_{22}^{-1}A_{21} & \hat{D} = D_B - C_2A_{22}^{-1}B_1 \end{array}\right]}_{Balanced\ SPA} \quad (22)$$

The proposed algorithm $(\hat{A}, \hat{B}, \hat{C}, \hat{D})$ is an outcome of the hybridisation of standard balanced truncation techniques with SPA to use the merits of both methods. Techniques have been used for a higher-order system, which is discussed in the numerical example section.

2.4. Error Analysis of Reduced Order Model:

If $G_r(s)$ is r^{th} order reduced-order model of the system, then modelling error transfer function is given by

$$E_r(s) = [G(s) - R(s)]$$

The actual (H-infinity) H_∞ norm error bound in the r^{th} order reduced model can be calculated by taking the infinity norm of the modelling error transfer function ($E_r(s)$). The actual and theoretical H_∞ norm error bounds are given by

$\|E_r(s)\|_\infty = \|[G(s) - R(s)]\|_\infty$ and $\leq 2\sum_{i=r+1}^{n} \sigma_i$ respectively. Moreover, the actual error bound is usually equal to or less than the theoretical error bound. It is also called the A priori error bound of the system (Antoulas, 2005).

Thus $\|E_r\|_\infty \le 2\sum_{i=r+1}^{n}\sigma_i$, an error bound is a measure of how near and close the reduced model is to the original system and determined based on either an additive $error\|G(s)-R(s)\|_\infty$ or a multiplicative $error\left\|\frac{G(s)-R(s)}{G(s)}\right\|_\infty$. Sometimes, the method of additive error is sufficient to give a reasonable reduced order model, but the multiplicative error approach appears to better fit on systems with slightly damped poles and zeros.

3. Numerical Experiments and Results

3.1. Example:

Let us consider a single-area power system model represented by following the 3rd order transfer function (Sonker *et al.*, 2017; Saxena *et al.*, 2013; Farid *et al.*, 2010; Tan, 2010; Tan, 2009) with a non-reheated turbine as

$$G_3(s)=\frac{250}{s^3+15.880s^2+42.460s+106.20}$$

$G_3(s)$ is written in the form of the state-space matrix

$$\left.\begin{array}{l} A=\begin{bmatrix}-15.880 & -42.460 & -106.20\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}, \\ B^T=[1\ \ 0\ \ 0], C=[0\ \ 0\ \ 250], D=0 \end{array}\right\}$$

$$\sigma(A)=[2.0694\quad 0.9222\quad 0.0299]$$

Calculate the Hankel singular values (HSV) of the dynamic system. The singular Hankel values measure each state's contribution to the input/output behaviour in state co-ordinates that equalise the input-to-state and output energy transfers. In addition, it indicates the desired and optimal order reduction of the original system. As per $\sigma(A)$, first to 2nd singular values are significant here and, consequently, singular values have deteriorated speedily. Therefore, the order of reduction has been preferred as a second order.

Then, finally, the 2nd reduced-order model is specified through this proposed approach (BSPA) as given by

$$R_2(s)=\frac{0.059720s^2-1.2460s+18.530}{s^2+2.6050s+7.8730}$$

which has a Hankel norm less than the sum of singular values $[\sigma_{r+1},\sigma_{r+2}\cdots,\sigma_n]$.

Figure 1. Step response of the reduced-order models with the original system

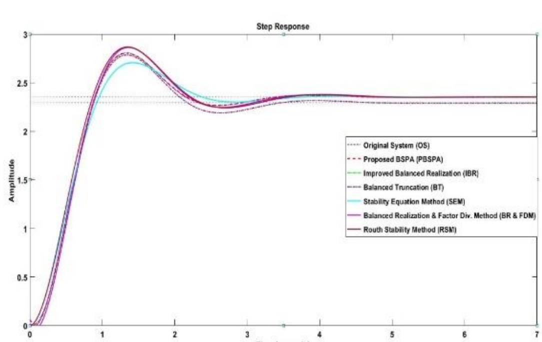

Figure 2. Step response of the reduced-order models with the original system for zoom

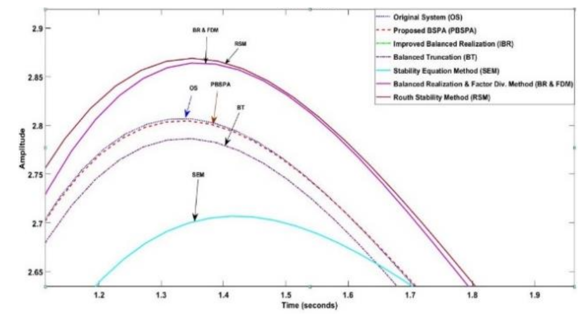

Figure 1 compared the step response of the ROM with the original system. It has been noted that it is a close approximation of the original system. Furthermore, another picturisation (zoom) of both systems response to an evaluation of the transient behaviour comparison is depicted in Figure 2. Again, it proves the ROM to be a near (close) depiction of the original system. Further, the measured the accuracy and closeness of the reduced-order model with the original system in terms of the H_∞ (H-infinity) norm. A modelling simulation error has also been calculated to show the effectiveness of the method respective to other existing methods, depicted in Table 1 and Table 3.

This is the validation of the proposed method. Note that the actual error-bound value is less than the error-bound theoretical values. The ROM is a close rapprochement of the system and compares its transient behaviour to the original system as well as other existing approaches, such as balanced truncation, improved BR and another composite approach.

Table 1: A Comparison based on H Infinity Norms for Example 1

MOR Approaches	ROM	H_∞ (H-infinity) Norm
Proposed Approach (PA)	$\frac{0.059720s^2-1.2460s+18.530}{s^2+2.6050s+7.8730}$	0.0597100
Balanced Truncation (BT) Method (Suman and Kumar, 2020b)	$\frac{-0.80560s+16.810}{s^2+2.3910s+7.3270}$	0.0597948
Composite of both (Balanced Realisation Method (BRM) and Stability Equation Method (SEM) (Suman and Kumar, 2020a)	$\frac{250}{s^2+2.6050s+7.8730}$	40.219115
Improved Balanced Realisation (IBR) (Prajapati and Prasad, 2019a)	$\frac{-1.26750s+17.2481}{s^2+2.3910s+7.3270}$	0.2639986
Balanced Realisation (BR) And Factor Division Method (FDM) (Mixed)(Prajapati and Prasad, 2018a)	$\frac{-1.26720s+17.2486}{s^2+2.3910s+7.3270}$	0.2638696
Improved Routh Stability Method (IRSM) (Prajapati and Prasad, 2019b)	$\frac{250}{15.880s^2+35.7720s+106.20}$	0.2465308
Improved Routh Approximation (Prajapati and Prasad, 2019b), Routh Approximation and Factor Division (Prajapati and Prasad, 2019c)	$\frac{24.0220s+8.6880}{s^2+13.3790s+7.9390}$	3.2106626
Pade Approximation (PA) Only (Saxena and Hote, 2013)	$\frac{-1.1910s+18.920}{s^2+2.708s+8.0430}$	0.1259567
Routh approximation (RA) Only (Saxena and Hote, 2013)	$\frac{18.680}{s^2+3.173s+7.940}$	0.6421096
Stability Equation Method (SEM) (Prajapati and Prasad, 2018a) and Pade Approximation Method (Chen *et al.*, 1980b)	$\frac{-0.00460s+249.9950}{15.880s^2+42.460s+106.20}$	0.4771010
Singular Perturbation Approximation (SPA) (K. V. Fernando and Nicholson, 1982b)(Kodra *et al.*, 2016)	$\frac{15.74}{s^2+2.674s+6.88}$	0.4834986
SEM (Chen *et al.*, 1979), FDM and Stability Method (Gupta *et al.*, 2018)(Sikander and Prasad, 2015a) Truncation Method (Smamash, 2007)	$\frac{250}{15.880s^2+42.460s+106.20}$	0.4771072

Routh Stability and Factor Division Method (N. Singh *et al.*, 2006)	$\frac{39.3660s+3344.70}{35.8s^2+584.80s+1420.80}$	1.9085503
Routh Stability and Pade Approximation (Pal, 1979)	$\frac{-15.7480s+249.995}{15.880s^2+35.7720s+106.20}$	0.4402325
Pade Approximation and Differentiation Method (D, 2013)	$\frac{-33.3210s+249.995}{2.2930s^2+28.307s+106.20}$	1.8151802
Factor Division Method (FDM) (Lucas, 1983)	$\frac{18.8170}{s^2+2.594s+7.9940}$	0.5342067
Routh- Pade Approximation (Prasad, 2000)	$\frac{24.022+8.6880}{s^2+13.3790s+7.9390}$	3.2106626
Routh Stability Method (Rao *et al.*, 1979)	$\frac{250}{15.88s^2+35.772s+106.20}$	0.2465308
Differential Method (Gutman *et al.*, 1982)	$\frac{250}{5.293s^2+28.307s+106.20}$	2.2237372

3.2. Example

Consider an example of 5th -order well-known transfer function of the stable linear system taken from (Prajapati and Prasad, 2018a)

$$\frac{10s^4+82s^3+264s^2+396s+156.00}{s^5+21s^4+84s^3+173s^2+148s+40.00}$$

$G_5(s)$ is written in the form of the state-space matrix

$$A=\begin{bmatrix}-10.50 & -42.00 & -86.50 & -74.00 & -20.00\\ 1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\end{bmatrix},$$

$$B=[1\ \ 0\ \ 0\ \ 0\ \ 0], C=[5\ \ 41\ \ 132.00\ \ 184.50\ \ 78],$$
$$D=0$$

$$\sigma(A)=[1.7347\quad 0.2171\quad 0.0134\quad 0.0075\quad 0.0042]$$

$\sigma(A)$, Calculate the Hankel singular values (HSV) of the dynamic system. The singular Hankel values measure each state's contribution to the input/output behaviour in state coordinates that equalise the input-to-state and output energy transfers. In addition, it indicates the desired and optimal order reduction of the original system. As per $\sigma(A)$, first to 2nd singular values (SV) are significant ones and, consequently, singular values have deteriorated speedily. So, the order of reduction has been preferred as a 2nd order.

Then, the 2nd reduced-order model is specified through this proposed approach as given by

$$R_2(s)=\frac{-0.0034470s^2+4.9550s+5.4010}{s^2+3.115s+1.3850}$$

Figure 3. Step response of the reduced-order models with the original system

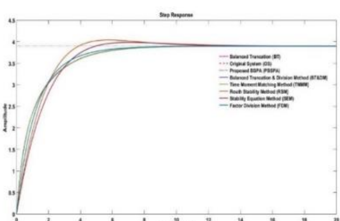

Figure 4. Step response of ROM with the original system for zoom

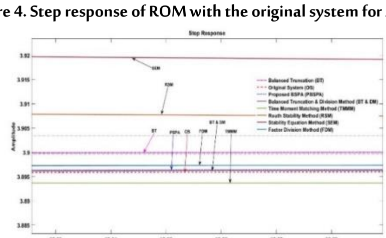

Figure 3 compared the step response of the reduced-order model with the original system. It has been noted that it is a close approximation of the original system. Furthermore, another picturisation (zoom) of both systems response to assessments of the transient behaviour comparison is also depicted in Figure 4. Again, it proves the ROM to be a near (close) depiction of the original system, providing another way to measure the accuracy and closeness of the reduced-order model with the original system in term of the H_∞ (H-infinity) norm.

Also, a modelling simulation error calculated to show the effectiveness of the method respected to other existing methods is depicted in Table 2 and Table 3. This is the validation of the proposed method and notes that the actual error-bound value is less than the error-bound theoretical values. The ROM is a close rapprochement of the system and compares its transient behaviour to the original system as well as other existing approaches, such as balanced truncation and another composite approach.

Table 2: A Comparison based on H Infinity Norms for example 2

MOR Method	Reduced Order Model (ROM)	H_∞ (H-infinity) Norm
Proposed method	$\frac{-0.0034470s^2+4.955s+5.4010}{s^2+3.1150s+1.3850}$	1.7764e-015
Balanced Truncation Method (Suman and Kumar, 2020b)	$\frac{10.061s+58.750}{s^2+19.25s+15.130}$	0.1003
Routh approximation (Prajapati and Prasad, 2019c)	$\frac{2.45510s+1.0340}{s^2+0.98470s+0.2662}$	0.7786
Balanced Truncation and Factor Division Methods (Prajapati and Prasad, 2018a)	$\frac{4.9380s+5.210}{s^2+3.0490s+1.336}$	0.0181
Balanced truncation (Lall *et al.*, 2002)(Sandberg and Rantzer, 2004)(Prajapati and Prasad, 2018a)	$\frac{4.923s+5.2150}{s^2+3.049s+1.3360}$	0.0166
Balanced residualisation (Prajapati and Prasad, 2019b)(Moore, 1981)(Zhou *et al.*, 1999)(Enns, 1984)	$\frac{-0.3450s^2+4.9550s+5.4001}{s^2+3.115s+1.385}$	0.3450
Stability equation and factor division Methods (Chen *et al.*, 1980a)(D. K. Sambariya and Prasad, 2013)(D. Sambariya and Arvind, 2016)	$\frac{369s+156}{167.997s^2+148s+40.00}$	0.9218
Routh approximation and Pade approximation (Lepschy and Viaro, 1982)(Shamash, 1975b)	$\frac{2.45480s+1.03822}{s^2+0.98470s+0.2662}$	0.7804
Pade approximation (Prasad, 2000)(Guillaume and Huard, 2000)	$\frac{4.64210s+4.37899}{s^2+2.6888s+1.112280}$	0.0717
Stability equation and Pade approximation methods (Shamash, 1975b)(Chen *et al.*, 1980b)(D. Sambariya and Arvind, 2016)	$\frac{369s+156.00}{167.997s^2+148s+40.00}$	0.9218
Differentiation method (Manohar and Sambariya, 2016)(Gutman *et al.*, 1982)	$\frac{5535s+9360.00}{1038s^2+3552s+2400.00}$	0.5703
Stability equation method (D. Sambariya and Arvind, 2016)	$\frac{369s+156.00}{167.997s^2+148s+40.00}$	0.9218
Routh stability (Krishnamurthy and Seshadri, 1978)(Rao *et al.*, 1979)	$\frac{310.6s+156}{128.2s^2+123.1s+40.00}$	0.8701
Modified factor division method (Lucas, 1986)(Parmar *et al.*, 2007)	$\frac{5s+5.1817}{s^2+3.1259s+1.3286}$	0.0630
Factor division (Prajapati and Prasad, 2018b) Pade approximation and modal methods (Shamash, 1975a)	$\frac{3.2475s+1.950}{s^2+1.50s+0.50}$	0.4626
Truncation method (Smamash, 2007)	$\frac{369s+156.00}{173s^2+148s+40.00}$	0.9640

Routh stability and Factor division methods (N. Singh *et al.*, 2006)	$\frac{271.90s+156.00}{128.2s^2+123.1s+40.00}$	0.9807
Differentiation method (Manohar and Sambariya, 2016)(Gutman *et al.*, 1982)	$\frac{5535s+9360}{1038s^2+3552s+2400}$	0.5703
Time moment matching (MMM) method (Zakian, 1973)	$\frac{4.1340s+3.90}{0.8906s^2+2.3947s+1.00}$	0.0717
Pade approximation and Differentiation Methods (D, 2013)	$\frac{1360.8s+9360}{1038s^2+3552s+2400}$	2.5617

Table 3. A Comparative Summary of Frequency Domain Computations on basis of H-infinity norm for Examples.

Computations	H_∞ (H-infinity) Norm					
	Example 1- 3rd Original Order System			Example 2 -5th Original Order System		
	BR & FDM Mixed	BT	Proposed Approach (PA)	BR & FDM Mixed	BT	Proposed Approach (PA)
$\lVert G(s) \rVert_\infty$ H-infinity	2.8496555			3.8996		
Theoretical H-infinity value $\leq 2 \sum_{i=r+1}^{2} \sigma_i$ For n=2 (ROM) A Priori Error Bound	0.0598			0.2914		
Actual H-infinity $\lVert G(s) - R(s) \rVert_\infty$ Error Bound in Simulation	0.2638696	0.0597948	0.05971000	0.01810	0.01660	1.7764e-015
Actual Relative Error Bound: $\frac{\lVert G(s)-R(s) \rVert_\infty}{\lVert G(s) \rVert_\infty}$	0.0926424	0.02098316	0.020953410	0.0046410	0.004256	4.5553e-016

4. Discussion

This article reveals the step response of the reduced model, and the original system is depicted in the figure above. All numerical experiments and Results have been carried out on the Intel ® CoreTM i7-8700 CPU @ 3.20 GHz and 8 GB memory using MATLAB R2019a (Academic Use) at the place of EED, MMMUT, Gorakhpur. The step responses of the original system and reduced-order model depicted in the figures of all examples are taken from the literature search. This figure shows that the reduced model is very close to the original system. This proposed method's excellence in comparison to the use of the BT method and other published works has been justified through two test systems. The H_∞ modelling error has been also computed and results are depicted in Tables. It is seen to be an excellent precise approximation with a minor error between the original system and ROM. It is observed that the results obtained by the proposed method are far superior.

5. Conclusion

A new reduced-order model approach for reducing the order of large-scale linear dynamic system has been proposed in this article. The proposed Balanced Singular Perturbation Approximation (BSPA) methodology is superior to any of the conventional methods or other mixed methods. Singular perturbation approximation from balanced truncation has reduced this process drawback. A BT and SPA approach to hybridisation has been used to effectively demonstrate a few examples of an LSLD system. Furthermore, the step response comparison shows that the ROM derived by the proposed approach offers a close approximation to the higher-order system. The responses are also compared in terms of an H-infinity norm. The proposed approach provides far superior results, which are justified by solving numerical examples taken from published work. This approach is more effective when successfully applied to a large-scale system. This procedure can be extended to the design of a state feedback controller, optimal, H- infinity controller etc.

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